

EVALUATION OF DETERMINISTIC CHAOS
IN AN ENERGY-ECONOMIC
DYNAMIC SYSTEMS MODEL

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Abstract

This thesis investigates the behaviour of a dynamic systems model which attempts to mirror the effect of society on demands for energy and the substitution of energy sources in the supply market.

Potential chaotic behaviour of the model was examined. Non-linear equations of the Energy Economic Dynamic Systems Model (EEDSM) were mathematically modelled. An attempt was made to cause bifurcation in each equation's results. The effect of ranges of equation coefficients were plotted. The plots were visually checked for bifurcation behaviour. None was found.

Bifurcation along with non-linear behaviour is accepted in the literature as a prerequisite for chaos. The author concluded that no chaotic behaviour existed at the level at which the system was examined.

Several adaptations were made to the equations, and ranges of sensible values were estimated for critical coefficients.

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1 Introduction

The Energy-Economic Systems Model (EEDSM) is a system which emulates global economic conditions and their prediction using energy concepts. Marchetti [1980] and others (see references) noted a prevalence of long wave behaviour in the trend of economic measures - where long-term trends endured regardless of short-term fluctuations. In pursuit of this the literature has examined various mathematical equations and their ability to mirror the long wave behaviour.

The EEDSM was Baines and Bodger's [1988] contribution to this effort. They reasoned (as did Marchetti [1977]) that energy use could be used as a barometer of economic activity. Not only is energy a fundamental requirement for every human activity, it is also straight forward to measure. Thus long-term data is readily available.

The model that they created (which is further examined in Section 2.2) compared favourably, in mirroring energy trends of the past and making future predictions of energy consumption, with the most popular economic equation - the logistic equation.

Having been based on a "systems approach" the EEDSM is a more intuitive representation of economic reality. It provides an insight into the structure of the system that it models rather than just reflecting its patterns. Coefficients in the model may be adjusted in an educated way. The model's behaviour is more amenable to coefficient-behaviour isolation, rather than the alternative "trial and error" approach that is necessary with the adjustment of economic equations.

Non linear systems may exhibit Chaos, which is extremely random and unpredictable behaviour. Logistic equations used by various authors in forecasting models have been found to have chaotic behaviour.

This project was created with the aim of determining if the EEDSM exhibited chaotic behaviour.

A further explanation of the EEDSM appears in Chapter 2, followed by an discussion on Deterministic Chaos in Chapter 3.

The methods of investigation are detailed in Chapter 4, including explanation of the preliminary work that helped to direct the research.

Chapter 5 reports on the research. It is divided into four parts: (1) a

general examination of the EEDSM, (2) simplification of the model as well as some adjustment to the equations, (3) investigating the critical equations, and (4) discussion of the findings.

Conclusions are presented in Chapter 6.

2 The Energy-Economic Dynamic System Model

2.1 The Advent of the EEDSM

Baines and Bodger [1984] contribution to the literature supported Marchetti's conceptual framework as a "useful alternative approach to the problems of energy demand forecasting and primary energy substitution."

Marchetti [1977, 1979, 1980] had examined the interactions between energy stocks and the social systems that they support. He noted that these interactions occur as logistic behaviour - as defined by

$$F_{\mu}(x) = \mu x (1-x) \quad (2.1)$$

the Logistic equation [Rasband 1990]. He also compared these interactions to that of a "learning society".

Baines and Bodger added to this two developments. The first was the concept of energy accessibility - a quantifiable measure of the ease of using the energy source. A higher accessibility value indicates that energy is easier to obtain from that resource.

Accessibility is a constituent part of a common representation of the energy requirements of energy: the energy yield ratio, or net energy yield.

An energy yield ratio is the ratio between the output of an energy supply industry and the cumulative energy requirements for accessing, processing and delivering that output, including material, capital and person-power inputs.

The addition of accessibility - "expressed in comparable energy yield ratios between competing sources at a point in time - may serve to fill the conceptual gap needed to explain in a physical way the actual introduction of a substitute energy form into the market" [Baines and Bodger 1984]. Accessibilities are examined by Odum in his 1976 paper "Net Benefits to Society from Alternative Energy Investments".

The second development addresses a hypothesis that Marchetti appears to disregard - that energy availability might affect the innovation necessary to make a new energy source viable.

Baines and Bodger proposed that the above is in fact the case. Although the capabilities of the new energy form initiate the work, it is the existing

energy form(s) that supplies the impetus for innovation that will enable the use of the new energy form. They illustrated this with the example of coal's substitution of wood.

In two papers Bodger and Tay [1986, 1987], examined several mathematical equations with respect to energy forecasting. They concluded that the Logistic equation was the best model for electricity consumption as a secondary energy form in the domestic, non-domestic excluding energy intensive industry (ND-EII) and combined markets.

The reasons for this finding were (1) that the Logistic model could be easily shaped to fit existing data, and (2) that its behaviour was bounded by asymptotes (rather than growing exponentially).

Bodger and Tay [1987] compared the Logistic model to an Energy Substitution Model (ESM). They concluded that the ESM [Baines and Bodger 1984] duplicates a substitution process of one energy form for another. Such substitutions appears to happen in the real world [Marchetti 1977]. The ESM was also found to be applicable for the substitution process of secondary energy consumption sectors.

However the Logistic model and ESM offered different forecasts of energy use. The Logistic model did not take other energy forms into account, whereas the ESM did.

Bodger and Tay concluded that more research was required to decide which forecast was the better.

In a 1989 paper Bodger and Baines proposed a third model with their Energy-Economic System. Forrester [1976, 1978] and Graham and Senge [1980] had produced an economic dynamic representation of society through equation and feedback loops. Their results appeared to show long wave behaviour in capital investment, production and capital stock.

The important change in method involved moving away from simply fitting equations to the recorded data, to that of creating a system with behaviour that mirrored the data. The author's hypothesised that forecasting with this system might be possible, and that some insight into the interacting processes of Energy Substitution might be gained.

Forrester (and others) saw the central long wave mechanisms arising from interactions between capital/consumer goods producing sectors. However Marchetti [1980] considered that the central mechanisms were related to the more basic working of society - the logistic functions of primary energy consumption, innovations and inventions and also these with respect to each

other.

However primary energy substitution patterns had still to be accounted for. Baines and Bodger [1984] had shown trends in New Zealand primary energy data that appeared to be moving away from the continuing logistic equation behaviour of Marchetti [1980] toward what they called a "stable, sustainable future characteristic of a damped dynamic system."

Society as a dynamic system has been examined by ecologists [Jackson, Davis 1979], and demonstrated for New Zealand society [Baines and Peet 1986]. The latter produced insight on variations in energy parameters, the net production of goods and services, and economic infrastructure for changes in availability of energy sources.

The environment or boundary conditions impinge strongly upon a dynamic system. Energy sources are important boundary conditions. For example any physical activity is strictly defined by available energy. Nothing in human society can happen without the use of energy of some form. Thus energy is a boundary condition that affects all facets of society, in many ways that money (say) can not.

Baines and Bodger extended this argument by stating that the Energy sources themselves are constrained by their availability and accessibility.

It would seem that a dynamic system does not need to be complex to represent society - as long as the constraining factors are addressed, the other parts of the process could be generalised.

2.2 A Dynamic Model of Society

To model society without the two constraints mentioned above would be a daunting task. Part of the problem is reduced by having a common basis for all the myriad facets of society - namely energy.

One does not need to study thermodynamics to realise that energy is not free. The only society that has free energy input (excluding solar), is equivalent to a group of animals. Fire, tools, crops, all require energy sources. The demands of many people living in one place quickly strip local resources that are supplied by nature, by chance. Effort is required to replenish these resources or to manage them.

Thus we have Figure 2.1(a), a simple relationship which can be qualified in two ways (1) society pays for the energy it receives, or (2) society buys the energy it wants. The price may be imposed or determined by market forces but it ultimately depends (as both the Resource Industry and Society depend) on

the resource availability and accessibility. Hence Figure 2.1(b).

These symbols represent a generalised view of each function. A more

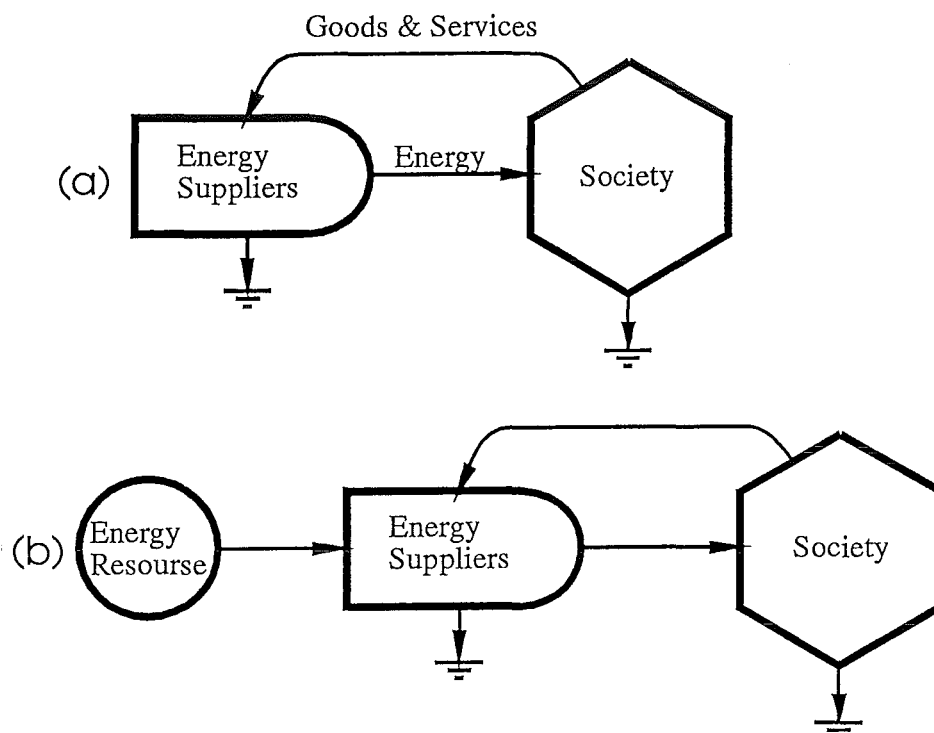


Figure 2.1. Simple views of Energy interaction (a) Suppliers - Consumer, (b) Resource - Exploiter - Consumer.

detailed view is examined in Section 5.1. We will presently restrict ourselves to a more superficial description of a complicated system - that of the New Zealand energy system, Figure 2.2 [Bodger, May 1992: Figure 4].

At the left of the figure we see that the resources have multiplied, as have the refining industries. All feed energy into Storage, which is the equivalent of distribution companies (coal merchants and petrol stations). The major energy storage is then fed into the Consumer/Producer sector which supplies manufactured goods and services - Feedback necessary for the extraction of the resource energy. The line labelled Direct represents sunlight, wind, small water-ways, thermal springs and the like. These bypass Storage entering the Consumer/Producer Sector directly.

The Energy Flow Source (bottom left) represents the growth of plants, rainfall and thermal activity. The Source also represents wind, sunlight (wind and solar farms) and wave action, as mentioned above. However these resources are not widely exploited in New Zealand.

These resources, in the real world, are in some cases seasonal. There is

no guaranteeing that a drought (say) would not reduce the resource. However the EEDSM iterates once for each year, reducing the affect of short-term effects.

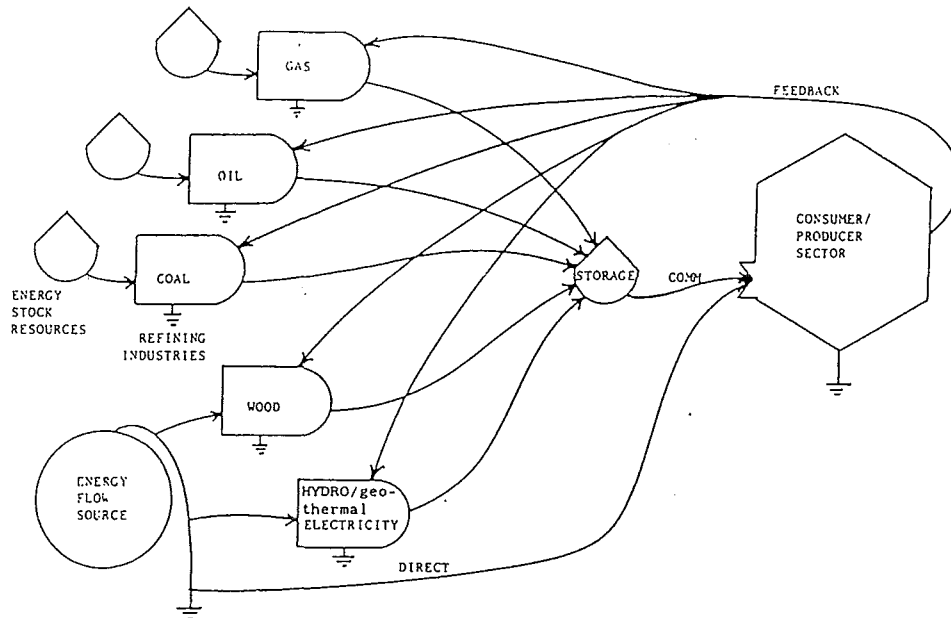


Figure 2.2. Dynamic model to represent the New Zealand energy system.

The resources in Figure 2.2 do not have equal market share. In the EEDSM, the historical share values have been mimicked by estimating the inception date, the year that each resource captured 0.01% of the market. At this time the resource becomes part of the market and competes with the existing resource(s).

The inception dates used are based mostly on Marchetti's[1980] values for the world, though the required technology or perceived need obviously took some time to arrive in New Zealand. Natural gas for example, was considered to be merely a by-product of oil extraction, until the first oil crisis in the early 1970s. It then quickly entered the energy market.

In Bodger and Baines [1988] the authors state that the important trend for their model to follow, is the long-term trend. The short-term fluctuations are scaled out of the behaviour curve for good reason - to mimic them may require a model which is as complex as the system itself. It was the long-term trends (not the short-term) that Marchetti[1980] compared to logistic curves, so it is those the EEDSM seeks to reflect.

For these reasons it is unnecessary for the model to be concerned with short term data. The model is concerned only with the year's totals, not the development of the totals.

In the model the results of the previous year are used to calculate the new year's totals. There are also internal calculations establishing the new size of the infrastructure of the Refining, Distribution, Producer and Consumer sectors. The size of the infrastructure determines the amount of energy transfer. Infrastructure is a build up of energy assets, used specifically for transferring energy. Part of the infrastructure will be expended in the transfer, and some energy in the form of goods and services will be added to it. The updating of the current infrastructure size involves feedback loops within the sectors.

Bodger and Baines[1988] noted that their model had generated "the major dynamic patterns of individual society (energy usage and industrial production)". They cautioned however that patterns were not exactly replicated.

They generated plots that showed that investment into each successive resource increased. This was said to be due to increased accessibilities, and society's increasing size. Society was prepared to spend money on a more efficient fuel - it could afford to spend more as it had more capital to invest - and it needed to create larger infrastructures to supply a larger pool of users.

The authors found that "price", which is not a part of the model, appeared "as a derived quantity and not as the determining factor of energy substitution." This proved energy was a secure common basis from which to model society.

Bodger and Hayes [1989] extended the EEDSM, by using it as a forecasting tool. The present trend is for gas to increase in market share, while wood, coal and oil continue to decrease. The EEDSM predicted that oil and the coal would return to dominance (after gas), followed in many decades time by gas again.

Nuclear power was added as a resource. It was found that if nuclear's accessibility equals gas's (the highest), nuclear will dominate after gas. If nuclear's accessibility is lower than gas, oil and coals (its energy yield ratio is lowest), it will eventually become market leader, but only after the present leader, gas, has been surpassed by both oil and coal.

Bodger and May [1992] rephrase these predictions. They discuss their difficulty in believing that hydro-electricity will suffer (as much as was predicted) from gas's dramatic dominance.

They suggest that unknowns may enter the system. Perhaps technological developments, may increase the accessibilities of wood, coal, oil

or electricity. Or an entirely new energy form may appear¹. The likelihood of re-introduction of previously discarded energy sources is reduced to be "in the very long term".

The EEDSM is a complex model comprised of twenty-two equations involving thirty coefficients. The equations of this model are examined in the next section.

2.3 The Equations of the Model²

2.3.1 The Energy Sector

The system of equations is represented by the following figure 2.3. It is a detailed view of the major parts of the previous figure 2.2

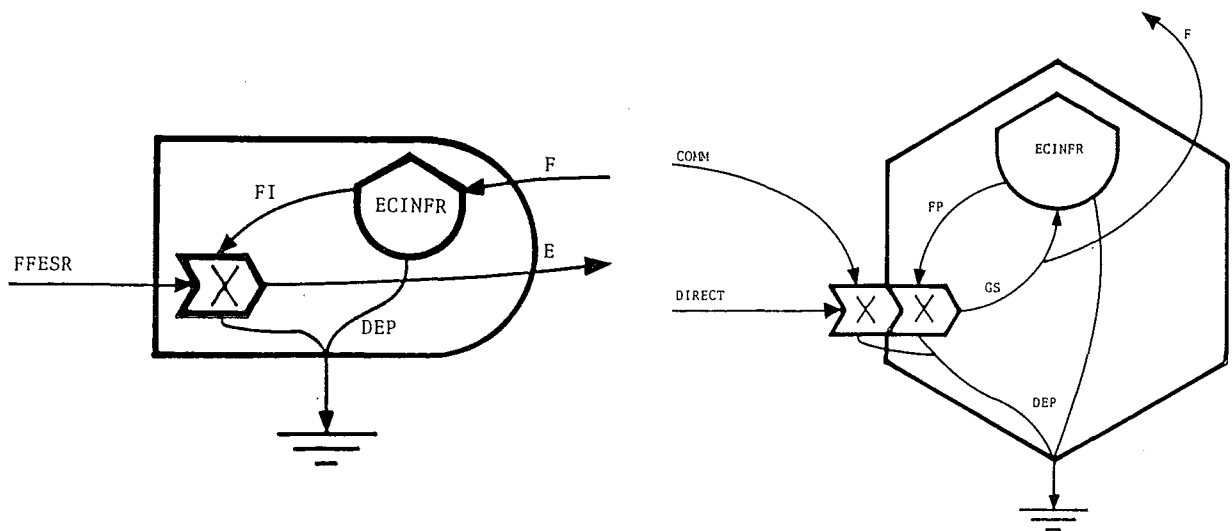


Figure 2.3 Visualizing the EEDSM, (a) representation of the energy-refining industries, (b) the producer-consumer sectors of society.

¹ An example of a possible new energy form is "clean" fusion.

² This section closely follows an unpublished paper by Bodger and Hayes, with permission from Dr. Bodger.

The flows into and out of an energy refining industry, v , are governed by the interaction between the size of the remaining stock resource, $R(v)$, and the accumulated effort to access the resource, $ECINFR(v)$.

The operational energy feedback $OPEFBK(v)$ is provided by the energy sector infrastructure to the workgate. It equals a constant, $H(v)$, times the infrastructure size $ECINFR(v)$, times the remaining stock resource for that sector, $R(v)$ as defined in Equation (1).

$$OPEFBK(v) = H(v) * ECINFR(v) * R(v)$$

EEDSM(1)

The feedback is proportional to the infrastructure, but it is also affected by the driving force behind the fuel flow from the untapped stock, $R(v)$. It can be thought of as the amount of water in a water storage tank with a pipe at the bottom. If $R(v)$ is large, then there is more pressure to force out the water from the tank, than if $R(v)$ is small. Hence the feedback is affected by the flow rate of fuel to the energy sector. This flow rate is directly affected by the pressure behind it, $R(v)$.

For a new energy sector, the energy output is initially set at 0.1% of the total energy output of all other energy sectors from the previous iteration. This is done to give the new sector a small but finite energy value relative to the other sectors at that time.

After the initial inception the energy produced by the refining sector (Equation 2) is the product of a constant, $G(v)$, the size of the sector infrastructure, $ECINFR(v)$, and the driving force or pressure of the fuel reservoir, $R(v)$.

$$E(v) = G(v) * ECINFR(v) * R(v)$$

EEDSM(2)

The Flow From the Environmental Stock Reserve, $FFESR$, to the refining industry is defined in Equation (3). It is a product of a constant, $K(v)$, the economic infrastructure of that sector, $ECINFR(v)$, and the remaining untapped stock resource for that sector, $R(v)$. The economic infrastructure supplies the workgate with the materials it needs (e.g. mining equipment, transport, personnel etc.). The more the workgate has at its disposal, the more fuel it can

access. More fuel will also be accessed if it is in plentiful supply, as opposed to being a scarce fuel. This is the dependence on $R(v)$.

$$FFESR(v) = K(v) * ECINFR(v) * R(v)$$

EEDSM(3)

The economic infrastructure, Equation (4), is a running total of its previous value and the present iteration value of the new energy sector infrastructure, $NEWINFREENERGY(v)$. This new infrastructure may be either positive or negative, depending on whether the sector is growing or decaying.

$$ECINFR_{j+1}(v) = ECINFR_j(v) + NEWINFREENERGY(v)$$

EEDSM(4)

The new infrastructure of the energy sector, $NEWINFREENERGY(v)$, equals the consumer feedback from the socio-economic sector, $CONSUMERFBK(v)$, less the operational energy sector feedback, $OPEFBK(v)$ and the depreciation from the infrastructure, $DEP(v)$. This is shown in Equation 5.

$$NEWINFREENERGY(v) = CONSUMERFBK(v) - OPEFBK(v) - DEP(v)$$

EEDSM(5)

It equals the input from the socio-economic sector less the outputs to the workgate and the environment. It is a balance of flows and is the growth or decline of the infrastructure.

The depreciation of an energy sector, $DEP(v)$, is a constant, $L(v)$, times the economic infrastructure of the energy sector.

$$DEP(v) = L(v) * ECINFR(v)$$

EEDSM(6)

It would be expected that the depreciation of a process, that is, the waste energy produced, is proportional to the size of the process itself, for a given efficiency.

The non-solar energy shares for each sector, $NONSOLARSHARE(v)$, are given by the amount of energy each sector produces, $E(v)$, divided by the sum of the energy from all the energy sectors. This is $COMM0$, or the sum of $E(v)$ as shown in Equation 7.

$$\text{COMMO} = \sum E(v)$$

EEDSM(7a)

$$\text{NONSOLARSHARE}(v) = E(v) / \text{COMMO}$$

EEDSM(7b)

Instead of looking at energy market share over time simply as a percentage, it is helpful, especially when working with curves of a logistic nature, to create a function of the form $F(x) = x/(1-x)$, and plot the log of $F(x)$ over time. This presentation is given by Equation (8) and is a derived pattern rather than one created by optimally fitting logistic curves. This is an important property of the model.

$$\text{FO}(v) = \text{NONSOLARSHARE}(v) / (1 - \text{NONSOLARSHARE}(v))$$

EEDSM(8)

The accessibility of an energy sector, $\text{ACCESS}(v)$, is the energy output, $E(v)$, divided by the operational energy feedback, $\text{OPEFBK}(v)$.

$$\text{ACCESS}(v) = E(v) / \text{OPEFBK}(v)$$

EEDSM(9)

It is a measure of the energy required to access a fuel and the energy the fuel will give out when used.

The 'average accessibility' is a consumption related weighted average of the accessibility of all energy sectors.

$$\text{ACCESSAVERAGE} = \sum (\text{ACCESS}(v) * \text{NONSOLARSHARE}(v))$$

EEDSM(10)

$\text{NONSOLARSHARE}(v)$ is always less than or equal to 1, and the average accessibility is influenced by the degree of market share each sector owns.

The quota of feedback goods and services from the socio-economic sector workgate to each energy sector is given by each sector's accessibility, its market share, and the average accessibility of all the energy sectors.

$$\text{QUOTA}(v) = \text{ACCESS}(v) * \text{NONSOLARSHARE}(v) / \text{ACCESSAVERAGE}$$

EEDSM(11)

The sector with the higher accessibility and market share, relative to another sector, will receive a greater percentage, or quota, of the feedback from the socio-economic sector.

The total new infrastructure in the energy supply sectors, NEWINFRTOT, is the sum of the individual new infrastructures in the energy supply sectors.

$$\text{NEWINFRTOT} = \sum \text{NEWINFREENERGY}(v)$$

EEDSM(12)

The total non-solar energy storage, NONSOLETOT, is a storage tank for the sum of the energy sector outputs, COMM0.

$$\text{NONSOLETOT}_{j+1} = \text{NONSOLETOT}_j + \text{COMM} - \text{COMMO}$$

EEDSM(13)

Its initial value does not affect the energy flow into the socio-economic sector. It acts as a damping factor to slow down the growth of the system dynamics, regulating the model from exponentially building up and driving itself into overflow., The larger the initial value of NONSOLETOT, the greater the damping effect. COMM0 is used as an energy flow. COMM is the value of COMMO from the previous iteration.

The remaining energy stock is the previous level less the present fuel flow from the stock reserve.

$$R(v)_{j+1} = R(v)_j - \text{FFESR}(v)$$

EEDSM(14)

2.3.2 The Socio-Economic Sector

The flows in the socio-economic sector are governed by the 'pressure' of the incoming energy, NONSOLETOT, and the accumulated effort to use it, ECINFRC.

The operating consumer feedback, OPCONSFBK, is the feedback in the socio-economic sector from the economic infrastructure to the workgate.

$$\text{OPCONSFBK} = \text{HH} * \text{NONSOLETOT} * \text{ECINFRC}$$

EEDSM(15)

This is given by the product of a constant, HH, the economic infrastructure of the socio-economic sector, ECINFRC, and the total non-solar energy output storage from the energy sectors into the workgate, NONSOLETOT. This is similar in structure to Equation (1), the operational energy feedback for an energy sector.

The depreciation of the consumer, or socio-economic sector, DEPC, is a constant, LL, times the magnitude of its economic infrastructure, ECINFRC.

$$DEPC = LL * ECINFRC$$

EEDSM(16)

The depreciation is directly proportional to the size of the infrastructure.

The gross output from the workgate of the consumer sector, GS, is the product of a constant, GG, the economic infrastructure, ECINFRC, and the non-solar energy output storage from both the stock sectors, NONSOLETOT.

$$GS = GG * NONSOLETOT * ECINFR$$

EEDSM(17)

It is another constant times the operating consumer feedback, OPCONSFBK (Equation (15)).

The total output from the consumer sector to the energy supply sectors, FTOT, is given by the gross output from the workgate of the consumer sector, GS, times a constant KF, divided by the average accessibility of all the energy sectors, ACCESSAVERAGE.

$$FTOT = KF * GS / ACCESSAVERAGE$$

EEDSM(18)

Some of GS is diverted to FTOT, the rest of GS is fed into the economic infrastructure. Therefore, FTOT must necessarily be smaller than GS, and acts as a negative feedback or moderator, for the energy supply sectors, via ACCESSAVERAGE. FTOT is proportional to the inverse of ACCESSAVERAGE, so if the average accessibility of the energy sectors is climbing, then FTOT will act to reduce the absolute amount of feedback they will receive, and vice versa.

The net consumer sector output, NETCOUTPUT, is the gross output, GS,

less the feedback to the energy sectors, FTOT.

$$\text{NETCOUTPUT} = \text{GS} - \text{FTOT}$$

EEDSM(19)

NETCOUTPUT is fed into the consumer sector infrastructure.

The new infrastructure in the consumer sector is the net gain or loss of the infrastructure, that is, the sum of the net consumer feedback to the workgate, OPCONSFBK, less the depreciation, DEPC.

$$\text{NEWINFRCONSUMER} = \text{NETCOUTPUT} - \text{OPCONSFBK} - \text{DEPC}$$

EEDSM(20)

The absolute consumer sector feedback to each of the energy sectors, CONSUMERFBK, equals the quota each sector commands, QUOTA(v), times the total amount of feedback available, FTOT.

$$\text{CONSUMERFBK}(v) = \text{QUOTA}(v) * \text{FTOT}$$

EEDSM(21)

The quota determine the relative feedbacks, while the total amount available determines the absolute feedbacks to the energy sectors.

The present economic infrastructure of the consumer sector is the previous infrastructure plus the new infrastructure, NEWINFRCONSUMER, which may be either positive or negative.

$$\text{ECINFRC}_{j+1} = \text{ECINFRC}_j + \text{NEWINFRCONSUMER}$$

EEDSM(22)

2.3.3 Constants and Initial Values

The constants used in the previous equations are set according to the initial values of the variables they involve. The form of the equations is a rearrangement of the appropriate equations in the set (1) to (22).

$$H(V) = \text{OPEFBK}(v) / (\text{ECINFRC}(v) * R(v))$$

from EEDSM(1)

$$G(v) = E(v) / (ECINFR(v) * R(v))$$

from EEDSM(2)

$$K(v) = FFESR(v) / (ECINFR(v) * R(v))$$

from EEDSM(3)

$$L(v) = DEP(v) / ECINFR(v)$$

from EEDSM(6)

$$HH = OPCONSFBK / (NONSOLETOT * ECINFRC)$$

from EEDSM(15)

$$LL = DEPC / ECINFRC$$

from EEDSM(16)

$$GG = GS / (NONSOLETOT * ECINFRC)$$

from EEDSM(17)

3 Deterministic Chaos

3.1 A Discussion of Deterministic Chaos

How is chaos related to the "real world"? The Oxford English Dictionary quotes chaos as 'a state resembling the formless void of primordial matter; utter confusion', and chaotic as 'utterly confused or disordered'. This suggests that nothing of value can be observed, that chaos is behaviour we are totally unaccustomed to.

However much we believe this, it is manifestly untrue. A hypothesis which will aid us in observing chaos is plainly needed. Chaos can be split into two classes, that which is (for the moment) truly formless, and that which is close to "normal behaviour". By normal behaviour I present the example of a fully deterministic system - given the systems constraining equation and initial values, every outcome is able to be predicted correctly (within a fine tolerance band).

This is what we could describe as a controllable system. The "real world" certainly encompasses these types of system, but they are a very small part of it. The real world is not strictly deterministic (if reality is to be completely described by dynamic equations, as above).

With regard to this, let us have Deterministic Chaos, an equation (say) with behaviour which is strictly deterministic for a few cycles and then is loosely deterministic. Let the tolerances be widened to allow results to be "near" these expected. So far we have achieved "approximation".

Approximation is the amount of prediction we are able to reliably sanction. Does the approximation, in itself, affect our "control" of the system? If the behaviour remains approximately predictable, our present approximation may be more correct than before, or, the tolerance band may be wider than that used previously.

How useful would a system be if it always remained inside a tolerance band? Is this simply an example of having such a wide tolerance band that the "predictive behaviour" becomes a meaningless term, a Ball-Park figure?

For this is the way a dynamically chaotic system may be predicted, by establishing a perimeter of behaviour. This prediction is at a price though, the position information is reduced to "being somewhere in the tolerance band". All other position information is lost! Is the former enough information to supplant the need for the latter?

The tolerance band may include unacceptable behaviour. This introduces the problem of knowing what may happen, without knowing how it will happen. Thus there is no possibility of preventing unacceptable system behaviour - there is no controllability.

Let's be practical. To have some knowledge of system behaviour, is obviously preferable to having none at all.

The alternative is to approach a problem blindly, perhaps to try every combination of equations to find behaviour tolerances that suit your system's requirements. Surely it is much more sensible and challenging to find some sense in the chaos. We will take this approach since it is reasonable to assume that knowledge of how dynamic systems develop could allow us some control.

An example is needed to show that some sense is to be found in a Chaotic System. The example is a simple form of chaotic attractor.

The following definition of attractors is from an article in Scientific American December 1986, by Crutchfield et al.

"Attractors are geometric forms that characterise long-term behaviour in the state space. Roughly speaking, an attractor is what the behaviour of a system settles down to, or is attracted to."

We will investigate what is called the "horse-shoe attractor" (Steward 1989). It is created using a series of two simple transformations, a stretch and a fold. These transformations are demonstrated in Figure 3.1. A rectangular (two dimensional) area is stretched to twice its length, while retaining its original volume. The new rectangle is then folded in half, fitting back into its

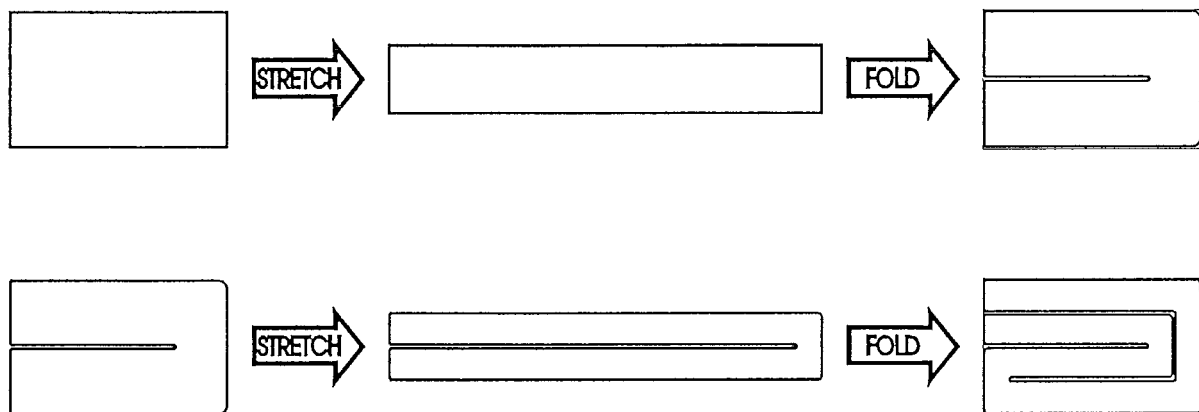


Figure 3.1. Creating a "horse-shoe attractor".

original shape.

This ideal version of the horse-shoe attractor allows one to imagine how an analytically simple system can appear random. In Figure 3.2(a) the system behaviour (which has been stretched and folded twice) has a cross travelling around the edge of the area. The cross moves at 4 units per second (the area is 2 by 4 units) and is drawn after each second. The travel time of the circumference increases from 3 seconds in Figure 3.2(a), until in Figure 3.2(c) it is 8.25 seconds.

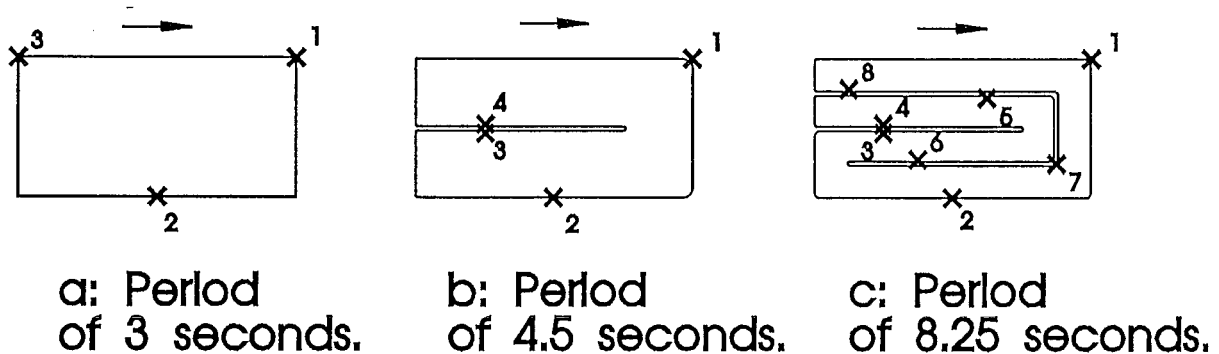


Figure 3.2. The behaviour of different horse-shoe attractors.

Note the path travelled around each figure. With only the crosses for information, little can be determined about system behaviour. Indeed any interpretation of system development from this information could be completely misleading. Perhaps rather than there being too little information, there may be too much information for us to make sense of.

A technique of determining the development of such systems will need to focus on a particular aspect, to remove the problem of excess information, or "noise". Ideally this aspect should be a feature of other dynamic systems.

One such detail was defined in Henri Poincaré's 1890 book, "On the problem of three bodies and the equations of dynamics" (in French) [Steward '89]. It detailed his attempt to find whether our solar system was stable. The title refers to the problem of solving the equations of motion for more than two bodies. With Newton's laws of motion such a calculation is considered impossible.

Poincaré reasons that a stable system is one that returns to its starting position, tracing out a closed curve as it moves. What is important in this hypothesis? Is it the path of the curve? Obviously this is important, but it in no way indicates periodicity. The only proof for periodicity is the curve passing through the initial point, with the initial velocity and direction.

The focus of Poincaré's study was periodicity and near periodicity. This

is a suitable aspect for us to also focus on.

A Poincaré Section (as the technique of recording periodicity is known) is simply:- At the point of interest, observe the plane orthogonal to the direction of travel. Record the position, velocity and direction of travel of each instance of the curve passing through the plane.

By using this method we filter out most of the system information, focusing attention on our one aspect. This automatically reduces both the computation and the comprehension requirements.

Observing the development of the section illustrates not only how this region of the attractor is formed, but further indicates the regions of the plane the system tends to occupy.

Initially periodicity is of most interest. Further familiarity leads to interest in pseudo-periodicity, as one begins to accept the attractor as a bounded region. This view allows the initially erratic behaviour, to have form. Acceptance, of the third trend "Intermittency" - the seemingly random return to periodic behaviour, is more likely than understanding it. Much of the behaviour is difficult to understand, which is why it is called chaotic.

Doubts abound. Can these generalisations hold throughout the system? Are Strange Attractors and Poincaré sections interdependent? What are their limitations?

Since the use of generalisation creates an atmosphere of doubt, we will naturally seek any form of additional proof available. Unfortunately many of these behaviours appear to exist, simply because they exist. This equation traces out this arc because that is its nature. This circular argument is unsettling. I hope to presently give some justification. Real-time development shows that the behaviour will eventually trace out a "slice" of the attractor shape. The attractor and section are obviously different representations of the same thing.

The real limitation is that the plane represents only two dimensions (the upper visual limitation is three dimensions). So we have to choose the view with some care, or luck.

One may require some reason to pursue the generalisation route any further.

Let us look at the thinking behind the process that we have followed. The starting point is a basic "law" of Physics. It is that ideal systems develop in a way that is completely predictable. By changing coefficients of such a system, it can be controlled.

From this "fact" a series of assumptions are made about the systems that exhibit dynamic chaos.

One: *A system shows (general) trends which can be observed.* There is some type of "connection" between initial and final conditions, ie. a rational or casual relationship.

Two: *Changing coefficients affects the trends.* This "connection" has a quality that a coefficient will have an effect on the system over time. This assumption is positive-time dependent.

Three: *Particular changes in coefficients can affect trends in a predictable way (generalised pseudo-determinism).* There is more than one connection, and there is some ability for differentiating between them.

Four: *The trends can suggest particular changes, in particular coefficients.* It appears that the connections can be individually identified. It may be that the connections are reversible.

Five: *Assumptions Three and Four describe a predictive situation equivalent to "control" of a system.* The observations can be categorised into different ways of exercising some control.

The behaviour of the known environment is, with these assumptions, extended into the unknown. In this case we wish to extend Deterministic Control into Chaotic Behaviour. Extending a trusted approach is an eminently reasonable first attempt. However the method of detecting behaviour is fundamentally different, as is the hypotheses that supports it as being rational. It is obvious that resultant theory will require new types of support (evidence, back-ground theory) for it to begin to explain the behaviour. It may be that such theory will be a higher level theory than basic Physics.

This theory should describe the observations made and through that explanation show how the data may be used to "control" the system. This information may need to be reduced³ to Physics (theory, laws) so that the system coefficients may be adjusted.

This could make Chaos theory a Meta-theory or higher theory - that which may be axiomatically based on accepted lower theory and thus can be reduced to and exhaustively explained by the lower theory and basic

³ Reduction:- is 'any doctrine that claims to reduce the apparently more sophisticated and complex to the less so'. This may involve either Physics based explanation, or be based partially in Physics and partially in higher theory. [Pan Reference '84]

observations of Physics.

The "handle" on the dynamics that we have gained at the meta-level is not simply axiomatic (as with the ideal analytical handles of deterministic systems). The insights are much more theory laden. The additional theory is in response to the method employed, describing the removal of data, and the observations and measurement of said data.

Generalisation removes all but a fraction of the system's diversity. The deleted data is not stored so that it can be reintroduced. But this should not be seen as a weakness. Stewart [1989] sums up this point nicely with the following passage on Lorenz's work in meteorology.

"If you look for the physics in Lorenz's equations, it's virtually non-existent. Better approximations to the true dynamics don't do anything like Lorenz's - as his colleagues pointed out to him at the time. Decades later one of them, Willem Malkus, said wryly: 'Of course, we completely missed the point. Ed wasn't thinking in terms of our Physics at all. He was thinking in terms of some generalised or abstracted model which exhibited behaviour that he intuitively felt was characteristic of some aspects of the external world!'."

The implication is that we have stepped from Physics to Mathematics - which is an abstract rendering of the real world. The techniques do not rely on the particular system, but properties which could be expected of all systems.

Another view is that the interpretation is that of an "ideal" case. Like any ideal case the meta-level view has no knowledge of abnormalities lying at its boundaries.

The meta-theory creates the methods which obtain observations, and then explains how these generalisations may be used to describe the entire system.

3.2 Sarkovskii's Theorem and the Period Doubling Route to Chaos

In this section a mathematical approach to chaos is examined. The description starts with the Sarkovskii theorem [Devaney '89]. This theorem describes how functions may describe the mapping of an interval (or point) onto an interval as below

$$I_1 \rightarrow I_2 \rightarrow I_3 \rightarrow I_4 \rightarrow \dots \rightarrow I_n \rightarrow I_1$$

That is each transformation is a result of the same function being applied

to each successive interval.

Note the final mapping returns to the original interval. After this point the transformations would repeat. A cycle of behaviour occurs.

If such a cycle occurs for a function then it is said to have a periodic point of "n" (the subscript of the final interval - the number of intervals in the cycle). Sarkovskii provides an ordering of natural numbers (below) and states that if n appears in this ordering then the function also has the periodic points for the numbers to the right of n.

Sarkovskii's ordering of natural numbers is:

$$3, 5, 7, \dots, 2^*3, 2^*5, \dots, 2^2*3, 2^3*5, \dots, 2^3, 2^2, 2, 1.$$

If a function F operates within Sarkovskii's scenario and that function has a periodic point of period 3, then it will also have periodic points with the period of every other natural number.

The most basic implication is that his function with period 3 "implies the existence of all other periods" [Devaney '89].

The second implication is that the period point contains information on the entire sequence of f^n - that knowing the period point p and the function f implies knowledge of the entire sequence of $f^n(p) = f(\dots f(f(p))\dots)$.

Before it can have infinitely many periodic points the function must first have all periods of the form 2^j .

There are two ways to achieve these 2^j points [Devaney '89]. One is period doubling⁴. This occurs for all unimodal maps (functions with a single mound shaped curve, typified by the squared term, as mentioned below).

In this form of function a periodic point can be found by plotting the function's curve, along with a $y = x$ line. Periodic points occur at the intersection of the two curves.

Figures 3.3 (a) & (b) are examples of this. The function $F_\mu = \mu x (1 - x)$ shown in figure 3.3 (a) has one period point, $x = 0$. Raising μ above a certain value doubles the number of periodic points of the function. This is period doubling. For any value of μ larger than 1 the function has two period points, see figure 3 (b). Hence $\mu = 1$ is a bifurcation point for F_μ .

If F_μ^2 is examined then the single maximum is replaced by two. As μ

⁴ The second way is through saddle node bifurcations - this is not covered here. Nor incidentally is it covered in Devaney's book.

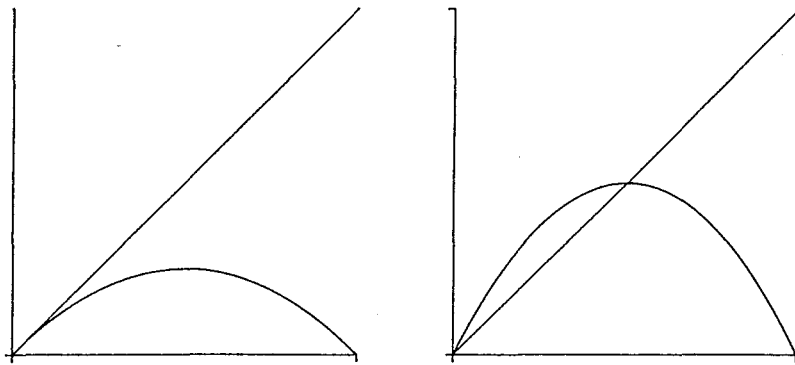


Figure 3.3. The graphs of $F_\mu(x) = \mu x (1-x)$ for (a) $\mu = 1$, (b) $\mu = 2$.

increases the maximums increase and the minimum decreases. Hence for the relationship $1 < \mu < 3$ there are two periodic points, which is figure 3.4 (a). When $\mu = 3$ there is a bifurcation point hence for $\mu > 3$ there are four period points, figure 3.4 (b).

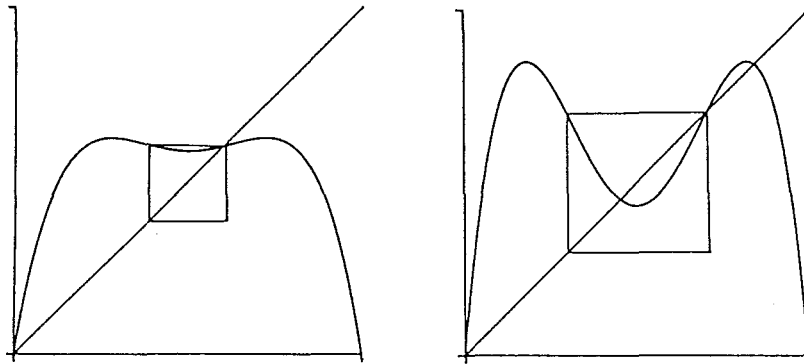


Figure 3.4. The graphs of F_μ^2 for (a) $\mu = 2.5$, (b) $\mu = 3.4$.

Repeating this process allows us to plot the vertical values of periodic points against the value of μ . This is shown in figure 3.5. This is a bifurcation diagram.

This graphical method suggests that the minimum requirement for a function to have chaotic behaviour is a square term in the equation(s). In the graphical representation of period doubling in Figures 3.3 and 3.4 the curves had two values of $y = 0$. This requires that regardless of the intermediate behaviour of the curve it must leave one zero point and return to another.

This relationship allows for a possibility of a minimum of two fixed points (the intersections with $y = x - x_1$, where x_1 is value of the first zero point).

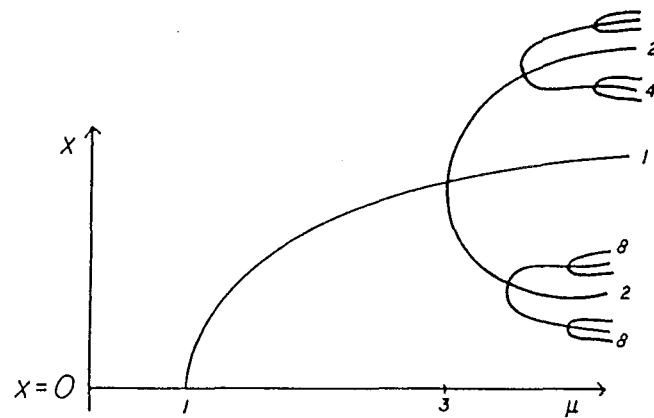


Figure 3.5. The bifurcation diagram for F_μ showing the repeated period doubling.

Any other curve will only ever have one fixed point, for as many interactions we might care to pursue.

Hence for the period doubling route to chaos, a square term for the independent variable x is a minimum requirement.

4 Detection of Chaotic Behaviour

4.1 Chaos that can be detected

Chaos is difficult to understand. Numerically it is a display of randomness. Visually, chaos begins to make some sense - the untrained eye can see patterns developing in it. It can even be beautiful.

Fractals are a familiar example of pictorial chaos. They are complex pictures, which can result from very simple equations. The detailed shape of a fractal will often contain smaller copies of itself. These copies are lodged in the edges of the shapes. Ian Stewart's book "Does God Play Dice" includes a pictorial journey into the Mandelbrot set. The picture is reduced 10^6 times, at which point the same basic shape is still prevalent. So an infinity of variety and detail can be expressed in a simple equation. As in the previous section understanding any of the levels, means that all the levels of detail are understood.

If a complex system's behaviour can be shown to be similar to a simple but chaotic model, then studying this model may lead to understanding of the system.

In Section 2.1 the equivalence of the logistic equation to long-term energy consumption behaviour was discussed. This equation is also said to exhibit chaotic behaviour [Steward 1989]. Triggering the chaotic behaviour of the logistic equation is simple and its development may be visualised.

The logistic equation is

$$y = \mu x (1 - x) \quad (4.1)$$

If we vary μ for this equation we would expect that there would be a different y as a result. If we kept the same μ , make $x = y$, recalculate, and then repeat these steps a hundred times, we could expect several possible outcomes. The numbers might increase to infinity, or decrease to negative infinity (as these are two distinct options). The numbers might jump around in a seemingly random manner. The numbers might begin jumping around a value, which they eventually converge to.

There is a further possibility. In Figure 4.1, for a value of μ between 3 and 3.4, the values of the resulting y 's converge, then jump between two values. For $3.4 \leq \mu \leq 3.5$ convergence is followed by a random run between four values.

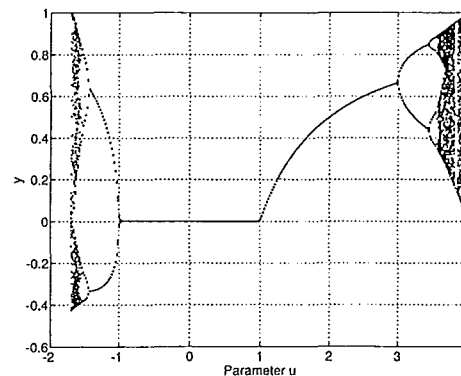


Figure 4.1. The Bifurcation diagram for the Logistic equation.

For higher values of μ , the result will alternate between 8, 16, 32, 64, 128, ..., 2^n values for y . This behaviour is illustrated in Figures 4.2(a) and (b).

Each value of μ that results in a doubling of values (hence period doubling) is called a bifurcation point. The entire plot is called a bifurcation diagram.

There is some dispute whether a bifurcation diagram is proof that the system is chaotic. Without rigorous mathematical proof, it can be said that for a fixed value of μ , there are a fixed number of possible y values, and that all these values can be calculated.

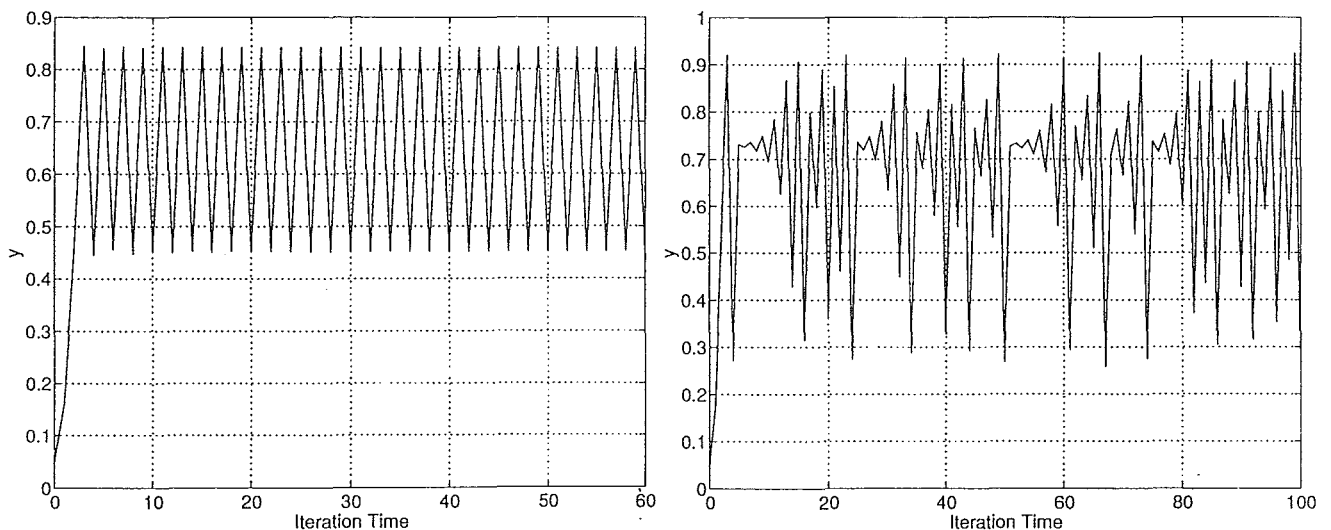


Figure 4.2. Time plots for the behaviour of equation 4.1 (a) at $\mu = 3.4$, and (b) $\mu = 3.7$.

Say that there are 128, or 2^7 , y values. Is there a pattern that these values are cycled through? If there is, the system is deterministic. If however the deterministic cycle is several thousand steps long, this knowledge may not be of much use. If a cycle is several million steps long then knowledge of it

may be of no use at all. From this view point the system may appear more random (chaotic), than deterministic.

There are two additional points worth noting about Figure 4.1. Firstly, $\mu = 1$ is a bifurcation point, except the values that would mirror those that appear above the y axis are all unstable and so are not real results (so are not plotted).

Secondly, there appears to be a gap in the chaos at $\mu = 3.84$. At this point there are three values which y alternates between. This is called the triple point [Steward 1989].

Recall the description in Chapter 3 of the stretching and folding behaviour of an attractor. The triple point represents the fold, where the behaviour evolves from the extreme of the stretch back to the origin.

The third point worth noting is that the detail of the behaviour repeats at different levels. Enlarging the behaviour framed by the ranges $3 \leq \mu \leq 4$ and $0.6 \leq y \leq 1$ (Figure 4.3), allows us a closer view of bifurcation and the triple point.

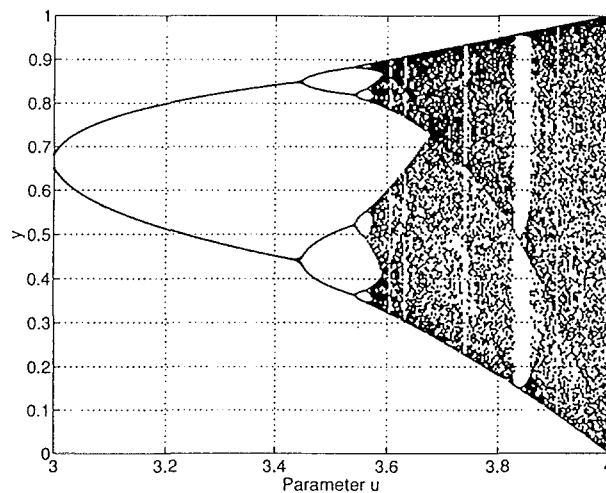


Figure 4.3. Enlargement of the Bifurcation diagram Figure 4.1.

The EEDSM model is a system of equations, which lends itself to examination by varying the coefficients (trying to create a bifurcation diagram).

To practice this approach the author examined a simple feedback equation described in equation 4.2, and Figure 4.4.

$$\begin{aligned} z &= \beta * y \\ y &= (c - z) \\ \text{or } y_{j+1} &= (c - \beta * y_j)^2 \end{aligned}$$

(4.2)

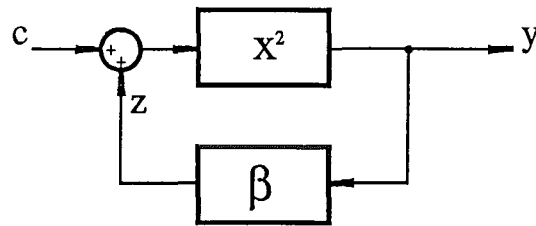


Figure 4.4. A flow diagram of the equation (4.2) tested for bifurcation. respectively.

A simple Matlab program was written which varied the coefficient β and input c individually (as the other was set to 1). The resulting values of y were then plotted against β , and c . These plots appear in Figure 4.5(a) and 4.5(b)

Two bifurcation diagrams are the result. Both seem to be asymptotically bounded in the positive range - y versus β , by $y = \beta - y$ versus c , c^2 . In all instances the bifurcation points occur at the same numerical value of β or c .

After further experimentation, it was found that the bifurcation points were occurring relative to the constant term. Regardless of whether c , or β , was set to be constant, an increase in the constant value caused the bifurcation points to shift towards zero (to the left on Figures 4.5(a),(b)).

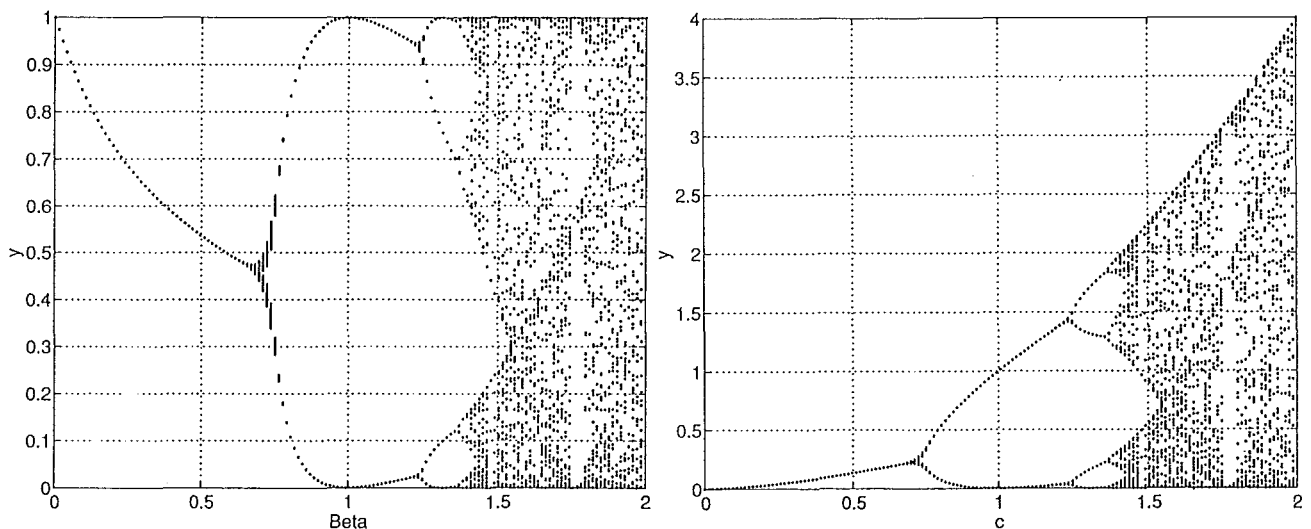


Figure 4.5 Bifurcation diagrams for equation 4.2, (a) y versus a varying coefficient β , (b) y versus a varying coefficient c .

The position of bifurcation points would appear to be dependent on every changing variable - for this case the behaviour is dependent on both the "degrees of freedom", the input value and negative feedback multiplier.

This raises a question that has serious consequences for any attempt of finding a bifurcation diagram for the EEDSM. The EEDSM has more than 20 equations, with over 30 coefficients. How many bifurcations diagrams might there be, and how will they effect each other? How can one tell if particular coefficients are grouped so that they only effect particular bifurcations?

These problems do help to suggest an approach - that is to somehow simplify the model to reduce the degrees of freedom.

4.2 The Packard Takens method:- A more holistic approach to detecting chaos

In Chapter 3 "strange attractors" were mentioned. They were described as being a multi dimensional limit band. A system that exhibits a strange attractor, will attract behaviour to be within the bounds of it. As was further demonstrated, using the horse-shoe attractor, position information is quickly lost. Prediction of the exact behaviour of the system is possible for a few seconds, a result which is usually of no practical use. A simple analogy for this is a dye in a fluid - you can tell where the dye is, but not where dye molecule number 2000 is!

In the previous section the EEDSM was said to have approximately thirty degrees of freedom, its behaviour having thirty changeable variables. If there is a strange attractor, this behaviour can be divided into regions in which motion is unbounded and regions in which motion is attracted into compact subsets. "These compact subsets are called attractors, the set of all phase space points which asymptotically tend to an attractor is called a basin of attraction." [Froehling et al. 1981]

A strange attractor for the EEDSM could have between three and thirty dimensions. Even if it were only three dimensional there is a problem - where in the thirty dimension phase space is the attractor?

A method which would remove this problem allows the representation of the system's chaotic attractor from data for any one part of the system. Although the EEDSM can supply data for all thirty-plus coefficients many experimental systems have only one data source.

The Packard-Takens method was proposed by Takens [Ruelle and Takens 1971] and developed by Packard [Packard 1980]. Takens supplied the

mathematical proof.

Packard-Takens, uses a single time series from one coefficient as the set of coordinates along the first (dimensional) axis for a plot in phase space. Depending on the dimension of the attractor, additional series of coordinates are created from the original time series. Four time series give coordinates for a four dimensional attractor (Strange attractors do not usually have a whole number dimension - one of the reasons that Packard-Takens supplies a representation of the attractor).

A delay between samples n is widely used. For the first series the original time series' first member $x(1)$ is sampled from the first coordinate. The n^{th} member $x(n)$ (where $n = 1, 2, 3, \dots, N = \text{series length}$) is taken as the second coordinate, the $2 * n^{\text{th}}$ as the third, until the required number of coordinates are gathered. The second set dimensional series starts from the second member $x(2)$ of the original time series, then $x(n+2)$, $x(2n+2)$, ...

If n is taken to be 1, then the time series is simply left shifted as shown in Figure 4.6.

```

101,22,73,143,5,62,117,...
    ↗ ↗ ↗ ↗ ↗ ↗
22,73,143,5,62,117,28,...
    ↗ ↗ ↗ ↗ ↗ ↗
73,143,5,62,117,28,295,..

```

Figure 4.6. Creating coordinate series.

There are three main problems with the method. The first is knowing whether the system has a chaotic attractor or not. The second is the delay time used in sampling the time series. The third is determining the embedding dimension n - the best number of dimensional-coordinate series to use in reconstructing the attractor.

Although establishing whether there is an attractor is probably the most important consideration, the Lyapunov method requires reconstruction of the system's attractor, that is the second and third problems affect the first.

Choosing the delay time τ (problem two) between samples, appears critical if τ equals a multiple of the period of the system attractor - this will reconstruct a plane through the attractor (effectively a Poincaré section). The data sampling time τ_s might innocently achieve this, or the addition of a delay k between samples might achieve it.

If τ is equal to any other value an infinite time series will allow complete

coverage of the attractor. However choosing τ too small or too large is a problem. If τ is too small then reconstructed points are almost the same and the attractor appears "stretched out along the $x = y = z = \dots$ direction [Froehling 1981]. Such small changes in data will also be affected by noise.

If the system attractor is chaotic, the "folding" part of the "stretch and fold" behaviour reduces the trajectory information, since one can only have orbital history for the length of the attractor period. If τ is much greater than the average time between folding, one to one correspondence with points of the time series and the original attractor is lost.

By 1985 Broomhead and King had proposed an iterated approach to determining the best τ , however it was still essentially trial and error. They also established an a priori estimate for n . They found that the coordinate window length

$$\tau_w = n * \tau \quad (4.3)$$

for each attractor point should be less than or equal to, $2\pi/w^*$ -the band-limiting frequency of the time series' Fourier spectrum.

The band-limit of a Fourier spectrum is the cutoff frequency where no greater frequency has significant power. For an estimate of n

$$n = \frac{2\pi}{w^* * \tau_L} \quad (4.4)$$

Liebert and Schuster's 1989 paper dealt solely with the selection of time delay

$$\tau = k * \tau_s \quad (4.5)$$

Their method involves calculation of the generalised Correlation Integral. When this is plotted against τ , for various embedding dimensions n , clear maximum and minimums are visible. The first minimums (with qualification - too complex to include here) indicates the best selection of τ . Although the method is computationally expensive, it does supply a quantifiable and repeatable

result.

The embedding dimension n has remained an iterated value. Its importance does not match that of other characteristics of behaviour, be they various other types of dimension or the correlation exponent. The information relating to these behavioural calculations improve as n is increased. Eventually there is no additional information gained by increasing n - this generally means that n is the best size for reproduction of a representation of the attractor.

However there are computational limits, as of Dvorák and Klaschka's 1990 paper that was up to an embedding dimension of 11.

If the system is chaotic (problem one) it will have at least one positive Lyapunov exponent [Shaw 1978, Bennett et al. 1976, Piesin 1976]. There are as many Lyapunov exponents as there are dimensions for a system. They measure the average rate of exponential convergence of trajectories onto the attractor if negative, and the average rate of exponential divergence of nearby trajectories within the attractor, when positive. The number of positive Lyapunov exponents gives a rough measure of the attractor's dimension, and the magnitude of them is a measure of the "degree of chaos". [Froehling et al. 1981]

Wolf, Swift, Swinney and Vastano published a paper in 1985 called "Determining Lyapunov Exponents from a Time Series" (At this time the Packard-Takens was still to be improved upon). Wolf et al., used their process to calculate the largest positive Lyapunov exponent (and second largest, where applicable) for several well known dynamic systems (sets of equations), and for experimental data.

The exponents units are "bits of information per second" (or "bits/orbit" for a continuous system, and "bits/iteration" for a discrete system), since they measure the amount of information the system creates or destroys. Wolf gives an example - the largest positive Lyapunov exponent of the Lorenz attractor 2.16 bits/second (for particular coefficient values). If an initial position was given with an accuracy of one part per million (20 bits), the subsequent position could only be predicted for ≈ 9.26 seconds.

Wolf's method involves reconstructing the attractor from a single time series, and examining the orbital divergence of small vectors over time. Initially the smallest vector is chosen (the closest two points on the attractor - not of the same period). One end is selected to represent the control trajectory (called the fiducial trajectory). The vector, length $L(t_0)$, is evolved around the attractor

and the length $L'(t_1)$, of the vector is recorded and assessed. The evolution time is chosen short enough that only small scale attractor structure is likely to be examined.

If $L'(t_1)$ is larger than desired, it will be replaced with a smaller vector made from the fiducial point and its nearest neighbouring point, where the resulting vector is less than a prescribed angle from the original (30° for Wolf et al.). If no suitable vector exists, or replacement is not necessary, the original vector is evolved and tested again.

When the entire data file has been traversed, λ_1 is estimated with equation 4.6

$$\lambda_1 = \frac{1}{t_M - t_0} \sum_{k=1}^M \frac{L'(t_k)}{L(t_{k-1})} \quad (4.6)$$

where M is the total number of replacement steps.

This approach was being examined by Dr. David Wall (Lecturer in the Mathematics Department, University of Canterbury), but was not available for the author's use, before the submission of this thesis.

5 Evaluating the Energy-Economic Dynamic Systems Model

5.1 Working out the workings of the Model

The person who best understands a model, such as the EEDSM, is the person who designs it. This poses a problem for the next person involved in the project, the learning curve is long, with deceptive turns. There is an advantage however in that this person will ask many awkward questions, will question the most basic assumptions and will not take much if anything for granted.

At the end of Section 4.1 simplification of the model was raised. The author's first task was to combine equations together in an attempt to end up with less equations. The author's first approach was to start with one of the EEDSM equations, and substitute equations for the coefficients. The equations which were substituted in were those which calculated the coefficients.

The following steps were to continue to substitute coefficient calculating equations until no further substitution was possible.

This stage was never reached. The problem had entered into infinite recursion - an expression which indicates that substitution continues without end.

The EEDSM program is started with arbitrary initial coefficient values - six per Energy Source and nine others. Thus the only possible outcome was to move from twenty-two equations with thirty coefficients to one equation with thirty coefficients.

This was obviously not an example of simplification.

The second attempt at simplification involved viewing the model as a collection of black boxes, one for each Energy Sector and one combining the Energy Store and the Socio-Economic Sector, Figure 5.1.

This greatly reduced the system's complexity since there were only three forms of interaction between these boxes :- (1) The energy leaving the Energy Sectors and entering the Socio-Economic Sector, (2) Feedback (energy goods and services) leaving the Socio-Economic Sector and entering the Energy Sectors, (3) waste energy leaving both the Energy Sectors and the Socio-Economic. Such a simplification requires that the equations inside the black boxes be combined into one

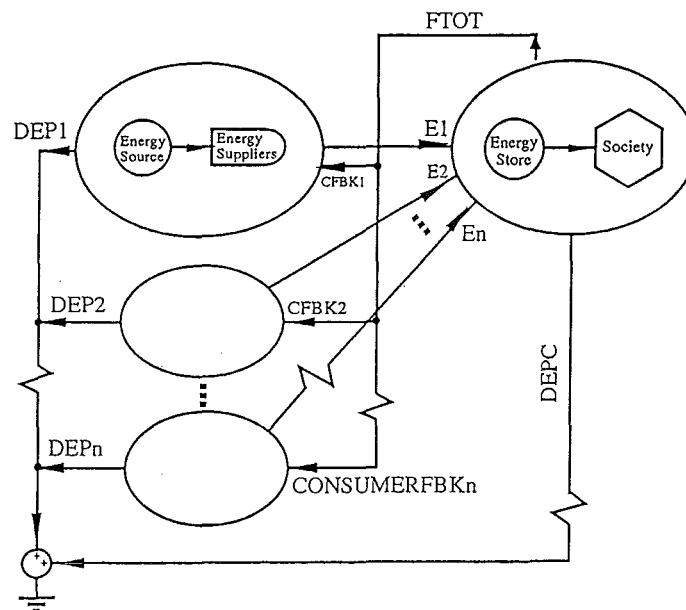


Figure 5.1. A simplification of the EEDSM - Energy Suppliers and Consumers.

equation per box.

Further study suggested that a third black box should be added separating the Energy storage system from the Socio-Economic. A second insight was that the inputs of each Sector do not simply come from the preceding Sector, as the diagrams would suggest, but also from the previous iteration.

There was more information flowing in the system than just the amount of energy transfer. We can see this in Figure 5.2.

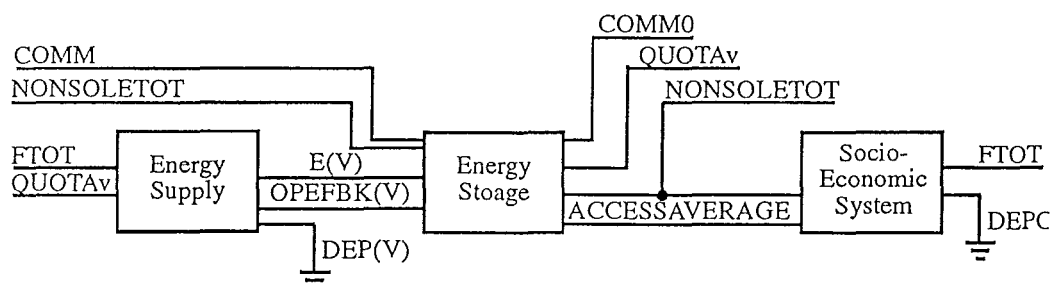


Figure 5.2. Another simplification of the EEDSM showing three Sectors with the quantities that flow between them (left to right).

Here it seemed that only four initial values were needed. Again the

black boxes were misleading since the Energy Sectors require six initial values each, and the Socio-Economic five initial values.

An alternative approach was considered. It involved simplifying the system to combine the coefficients of various equations together. The only representation of the entire system were the Figures 4 and 5 in Bodger and Baines 1989, and Figure 4 from Bodger and May 1992. These Figures are shown below in Figure 5.3(a),(b), and Figure 2.2 respectively.

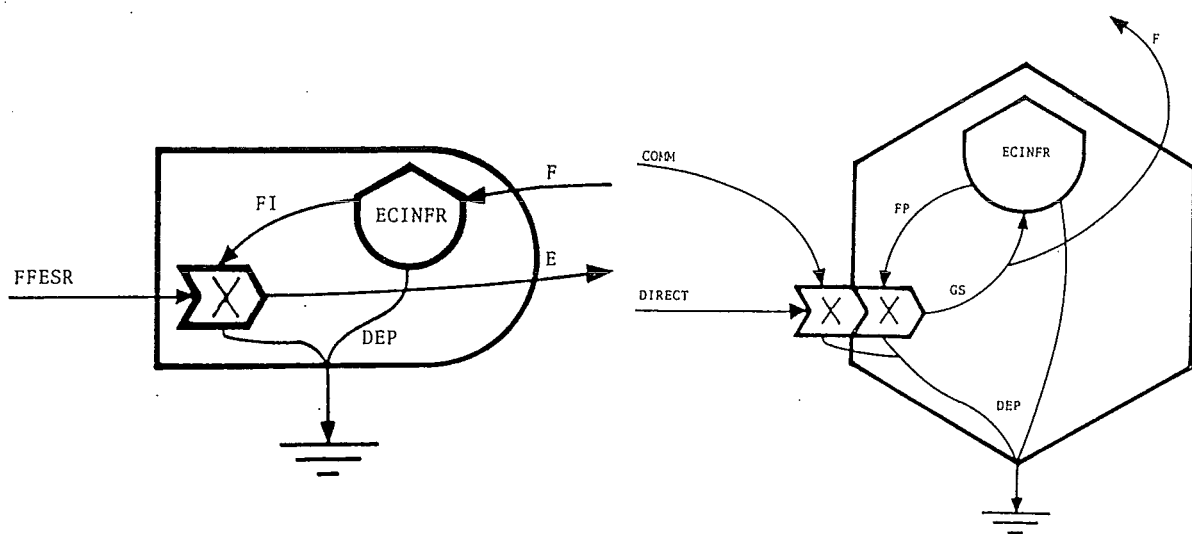


Figure 5.3. Visualising the EEDSM, (a) representation of the energy-refining industries, (b) the producer-consumer sectors of society.

The author interpreted figure 5.3 (b) directly into the flow diagram in figure 5.3(c). The interpretation involved redrawing figure 5.3 (b) as a flow diagram. Here a b $1-b$ c d and $1-d$ represent the two segments of the workgate. a is the proportion of energy flow from Direct plus Comm (x_0). b is the proportion of this energy which becomes waste from the first segment of the workgate - therefore $1-b$ is the proportion left.

The proportion $1-b$ is added to the feedback from $1-f$ (corresponding to FP which comes from $ECINFR$ in figure 5.3(b)) and a proportion of this sum c continues. d is the proportion of energy which becomes waste from the second segment of the workgate - leaving $1-d$ to continue (GS in figure 5.3(b)).

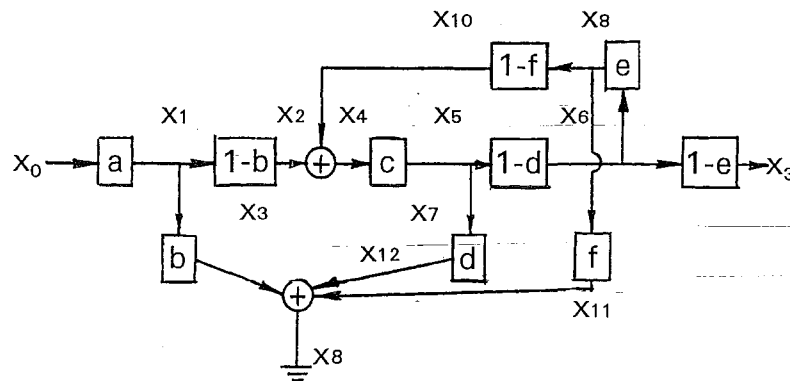


Figure 5.3(c) A flow diagram representation of Figure 5.3(b)

In this flow diagram x_n represents an energy flow value, and the letters represent coefficients which multiply with the flow. The coefficients are all in the range $0 < \text{coefficient} < 1$. The coefficients are positive because negative energy flow and creating energy (without a greater energy input) is not possible. At each of the branches a proportion of the energy flow is drawn off f and the remaining flow equals the original multiplied by 1 minus the coefficient.

Rather than writing equations from this flow diagram, the author began to simplify it. The interesting aspect of the diagram is the feedback loop consisting of e and $1-f$. This is where non-linear behaviour would arise. The waste energy can be ignored because the removal of the waste is given by the $1-b$, $1-d$, and $1-f$ coefficients. Hence removing the waste energy parts of the flow diagram will not affect the result. Figure 5.4 continues to have x_9 as output.

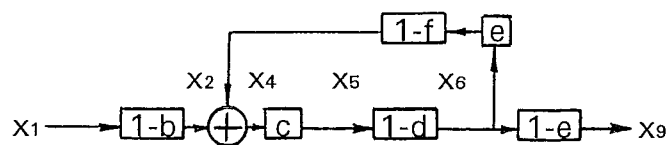


Figure 5.4 Initial simplification of figure 5.3(c) by removal of the coefficients which calculate the proportion of waste energy.

The next simplification concentrates further on the feedback loop. The two coefficients $1-b$ and $1-e$ are removed. Notice that in figure 5.5 the input and output have changed in response. The series terms c $1-d$ and $1-f$ e have been combined by multiplying them together.

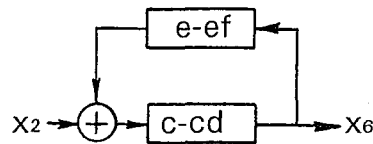


Figure 5.5 Further simplification of the flow diagram by concentrating on the feedback loop.

With a method from DiStefano [1976] this feedback equation was simplified into approximate linear¹ form which appears below.

$$\frac{c(1-d)}{1-e(1-f)c(1-d)} \quad (5.1)$$

This simplified form was however inconsistent with the equations on one major point. There are not three coefficients for calculating the depreciation (b, d, and f), but one. This suggested that the diagram did not correctly represent the equations.

Although this attempt failed the original approach still seemed rigorous and so an attempt was made to construct a more correct diagram from the equations. The resulting flow diagrams (Figures 5.6(a),(b),(c)) appeared to be less visually acceptable. Perhaps this was because they showed all of the details of the equations, rather than the essential information presented by the originals. In this way they lost the generalised appearance of a system.

Of more importance however, the new diagrams suggested some discontinuity of logic in the creation of the equations.

The most obvious problem was with equation (16) of the EEDSM

$$DEPC = LL * ECINFRC$$

EEDSM(16)

¹ The linear equation implies no possible chaotic behaviour. However this is only a artefact of the method. The major conclusion to be drawn from this experiment is the misrepresentation of the EEDSM equations by Figure 5.3 (a) & (b).

Socio-Economic Waste energy = constant * Economic Infrastructure

In this form DEPC represents the amount of energy stored in the Economic Infrastructure that would be wasted. It however seemed to ignore the loss that must occur in the transfer of NONSOLETOT energy into tangible goods and services - having units of energy.

Equation (20) of the EEDSM

$$\text{NEWINFRCONSUMER} = \text{NETCOUTPUT} - \text{OPCONSFBK} - \text{DEPC} \quad \text{EEDSM(20)}$$

New Soc-Eco Infrastructure =

$$\begin{aligned} & (\text{Goods \& Services Produced} - \text{Energy Sector Feedback}) \\ & - (\text{Soc-Eco Feedback}) - \text{Soc-Eco infrastructure decay.} \end{aligned}$$

lent further strength to this belief since all of the energy was transferred into something useful. What was not available for the infrastructure was used to "pay" for energy or consumed in the production process.

Some wastage must have occurred during production from machine inefficiencies and production error. The DEPC is a measure of the "decay" of the ECINFRC, it could not claim to also account for process waste.

The problem was solved by the addition of a second depreciation affecting the NONSOLETOT.

$$\text{DEPC}_1 = \text{LL}_1 * \text{ECINFRC} \quad (5.2)$$

$$\text{DEPC}_2 = \text{LL}_2 * \text{ECINFRC} * \text{NONSOLETOT} \quad (5.3)$$

which alters EEDSM(19) and (20) respectively

$$\text{NETCOUTPUT} = \text{GS} - \text{FTOT} - \text{DEPC}_1 \quad (5.4)$$

$$\text{NEWINFRCONSUMER} = \text{NETCOUTPUT} - \text{OPCONSFBK} - \text{DEPC}_2 \quad (5.5)$$

The problem above, pointed to the EEDSM equation (6)

$$\text{DEP}(v) = \text{L}(v) * \text{ECINFR}(v) \quad \text{EEDSM(6)}$$

Here the problem was the same, and was solved by including a depreciation of $R(V)$ - the Energy Source.

$$DEP_1(v) = L_1(v) * ECINFR(v) \quad (5.6)$$

$$DEP_2(v) = L_2(v) * ECINFR(v) * R(v) \quad (5.7)$$

There was no equivalent equation in the Energy Storage Sector.

The diagrams in Figures 5.6(a),(b),(c) were obtained through simplification of the initial flow diagrams. Examining these diagrams, it can be seen that the relevant part of the diagrams are used to calculate the required output. In essence this means that a number of equations are combined together, resulting in a reduced number of equations - one for each output (or internal feedback).

Hence for the Energy Supply Sector, Figure 5.6(a), there were three output equations and two internal feedback equations. For the Energy Storage Sector, 5.6(b), there were four output equations. For the Socio-Economic Sector, 5.6(c), there were four output equations (including $DEPC_1$ and $DEPC_2$) and one internal feedback equation.

It seemed that a form of model simplification had been achieved.

5.2 The simplified model

The basic requirements of Chaotic behaviour were then examined. The simplest was that the system had to show some non-linear behaviour. This made the simplification problem even easier, since the only equations in the system that were non-linear were the two feedback equations in the Energy Supply Sector for $ECINFR(V)$ and $R(V)$, the $NONSOLETOT$ equation from the Energy Storage Sector, and the feedback equation for $ECINFRC$ in the Socio-Economic Sector.

These five equations were made by combining several EEDSM equations, as mentioned in the previous section.

There were two equations from the Energy supply sector. The first equation calculates the new infrastructure for an Energy Sector. One equation would be required for each energy resource.

The equation is EEDSM equation (4)

$$ECINFR_{j+1}(v) = ECINFR_j(v) + NEWINFREENERGY$$

EEDSM(4)

with EEDSM(1), EEDSM(5), 5.5, 5.6, and EEDSM(21) substituted for the final term, resulting in equation 5.8.

$$ECINFR_{j+1}(v) = ECINFR_j(v) * (1 - L_1(v) - ((H(v) + L_2(v)) * R_j(v))) + (FTOT + QUOTA_v)$$

(5.8)

$$\begin{aligned} \text{New Energy Sector Infrastructure} = \\ \text{Infrastructure} * (1 - c1 - ((c2 + c3) * \text{Resource remaining})) \\ + (\text{Feedback for Energy Sectors} * \text{Quota}_{\text{Sector}}) \end{aligned}$$

where V represents a particular energy source (Sector), $c1$ is the coefficient used in calculating DEP_1 - the decay of the sector infrastructure, $c2$ is the coefficient in the equation for $OPEFBK(V)$ - the operating feedback for the sector, and $c3$ is the coefficient used in calculating $DEP(V)$ - the waste energy of the sector.

The second equation calculates the new Resource reserve for an Energy Sector. Again one equation would be required for each sector.

The equation is EEDSM equation (14)

$$R_{j+1}(v) = R_j(v) + FFESR(v)$$

EEDSM (14)

with EEDSM(3) substituted for the final term, resulting in

$$R_{j+1}(v) = R_j(v) * (1 - (K(v) * ECINFR_j(v)))$$

(5.9)

$$\begin{aligned} \text{New Resource Size} = \\ \text{Resource} * (1 - (c4 * \text{Energy Sector Infrastructure})) \end{aligned}$$

where $c4$ is the coefficient in the equation for $FFESR(V)$ - the fuel flow into the Energy Supply Sector.

The equation from the Energy Storage Sector calculates the size of the Non-Solar Total energy storage. It is the EEDSM equation (13)

$$\text{NONSOLETOT}_{j+1} = \text{NONSOLETOT}_j + \text{COMM} - \text{COMMO} \quad \text{EEDSM(13)}$$

New Non-Solar Total =

$$\text{Non-Solar Total} + \text{previous total Energy output} \\ - \text{total Energy output}$$

The equation's purpose was not simply to calculate the total energy, but to determine how much energy would be offered to the Socio-Economic Sector in each iteration. In this way EEDSM(13) acts as a governor, keeping the entire system set to a certain level of performance.

This equation was unique in being part of the current iteration. The other equations 5.7, 5.9 and 5.10 which follows, are equations which adjust the system's infrastructure affecting the next iteration.

There was one non linear equation from the Socio-Economic Sector. The equation calculated the new infrastructure for the Socio-Economic Sector.

The equation is EEDSM equation (22)

$$\text{ECINFRC}_{j+1} = \text{ECINFRC} + \text{NEWINFRCONSUMER} \quad \text{EEDSM(22)}$$

With EEDSM(15), 5.5, 5.6, EEDSM(19) and 5.4 substituted for the final term, it results in

$$\text{ECINFRC}_{j+1} = \text{ECINFRC}_j * (1 - \text{LL}_1 + \text{NONSOLETOT} \\ * (-\text{LL}_2 - \text{HH GG} * \\ * (1 - \text{KF/ACCESSAVERAGE}))) \quad (5.10)$$

New Soc-Eco Infrastructure (SEI) =

$$\text{SEI} * (1 - \text{c5} + \text{Non-Solar Energy} \\ * (-\text{c6} - \text{c7} + \text{c8} \\ * (1 - (\text{c9/Energies' average accessibility}))))$$

where c5 is the coefficient used for calculating DEP_1 - the decay of the sector infrastructure, c6 is the coefficient used in calculating OPCONSFBK - the energy feed into the Soc-Eco Infrastructure, c7 is the

coefficient in the equation for $DEPC_2$ - the waste production energy of the sector, c_8 is the coefficient in the equation for GS - the production of goods and services, and c_9 is the coefficient used in the equation for FTOT - the feedback to the Energy Sectors.

With respect to the Socio-Economic Sector equation 5.10, the equation has five coefficients, and three variables. The reaction of the equation to changing values for variables, was to be studied. As for the coefficients, the author's aim was to leave each one at set a value. However these values were not known.

The alterations which were made to the Socio Economic system's equations (section 5.1) suggest a particular relationship between GS, OPCONFBK and $DEPC_2$. These three components account for the entire energy flow through the Socio Economic infrastructure (from NONSOLETOT).

The coefficients GG, HH and LL should be seen as having a dimension of 1/energy. The coefficients should either sum to one, or add up to another coefficient (of the author's invention) KK. This value KK would be used as a coefficient for an equation similar to that for FFESR(V) (EEDSM(3)) in the Energy Supply Sector. KK would combine with ECINFRC to calculate a value for the full amount of energy passed through the ECINFRC, as in equation 5.11

$$\begin{aligned} \text{Energy Input} &= KK * ECINFRC * NONSOLETOT \\ &= GS + OPCONFBK + DEPC_2 \end{aligned} \tag{5.11}$$

The relation of the coefficients

$$KK = GG + HH + LL_2 (=1 : \text{ideal case}) \tag{5.12}$$

is not mentioned in the papers describing the EEDSM. It must be assumed that this was not the relationship that was used. Yet it is an intuitive relationship which helps to define the coefficients and the proportions of GS, OPCONFBK, and $DEPC_2$.

Before testing the equations for bifurcating behaviour (as in Section 4.1), the author wished to estimate the ranges of coefficients. If

the range could be known without need to refer to the rest of the system or arbitrarily setting initial values, this would reduce the effect of the coefficients on the equation's behaviour.

The method of establishing coefficient values seemed different to that of Bodger and Baines, the investigation would not be of the EEDSM (as it were), but of a new version of it.

As Bodger and Baines's method of setting the coefficient values could not be deduced and parts of their EEDSM had already been assumed to be incorrect, the author decided to alter the equations and methods.

If the equation 5.11 were accepted, then the equation for GS, EEDSM(17) could be seen as

$$GS = KK * NONSOLETOT * ECINFRC - OPCONSFBK - DEPC_2 \quad (5.13)$$

Goods & Services Produced =

Input energy - Feedback - Waste energy

where $KK = 1$.

If equation 5.12 as well as EEDSM's equation (19) were substituted into EEDSM(20), the following would result

$$\begin{aligned} \text{NEWINFRCONSUMER} &= (GS - FTOT - DEPC_1) - OPCONSFBK - DEPC_2 \\ &= ((KK * NONSOLETOT * ECINFRC - OPCONSFBK - DEPC_2) \\ &\quad - FTOT - DEPC_1) - OPCONSFBK - DEPC_2 \end{aligned} \quad (5.14)$$

The situation was obviously incorrect - OPCONSFBK and $DEPC_2$ reduced the total input energy leaving GS, as in equation 5.13, and then reduced NETCOUTPUT as in 5.14.

The answer was to change equation EEDSM(20), to remove OPCONSFBK and $DEPC_2$. This made EEDSM(20) unnecessary, and EEDSM(22) changed to

$$ECINFRC_{j+1} = ECINFRC_j + \text{NETCOUTPUT} \quad (5.15)$$

and with EEDSM(15), 5.2 and 5.4 substituted into 5.15

$$ECINFR_{j+1} = ECINFR_j * (1 - LL_1 - NONSOLETOT * (GG * (1 - KF/ACCESSAVERAGE))) \quad (5.16)$$

The effect of the altered equation is seen in the flow diagram of Figure 5.5(b).

The flow diagram for the Energy Supply Sectors Figure 5.4(a) shows the same relationship for the division of the inward energy flow as was seen for the Socio-Economic Sector.

At first glance it would seem that the total energy input was divided into OPEFBK(V) - the operational feedback for the sector, E(V) - the Energy supplied as output, DEP₂(V) - wasted energy, and FFESR(V) - the Flow From the Environmental Stock Reserve.

This variable FFESR(V), was the total amount of energy entering the system. It should equal the sum of E(V), DEP₂(V), and OPEFBK(V). However the description of OPEFBK(V) from Bodger and Hayes unpublished, showed that none of the "work-gate output is... fed back into the refining sector economic infrastructure." The equation EEDSM(1) for OPEFBK(V) simply uses the amount of energy flow to calculate the feedback required for the present iteration - the energy came from that stored in the economic infrastructure, ECINFR(V).

So now the input energy relationship was

$$FFESR(v) = E(v) + DEP_2(v) \quad (5.17)$$

There was also a DEP₁(V) the decay of the infrastructure. The infrastructure has units of energy. The energy was either stored in the ECINFR(V), or input from CONSUMERFBK(V) - the proportion of Socio-Economic feedback given to this Energy Sector (proportionally to the Energy's Market Share).

The DEP₁(V) does not seem to take the inflow of consumer feedback into account, however it would be better to have one measure of waste energy for both of these sources. It was adequate to assume that all the consumer feedback arrived intact - and that the equation 5.6

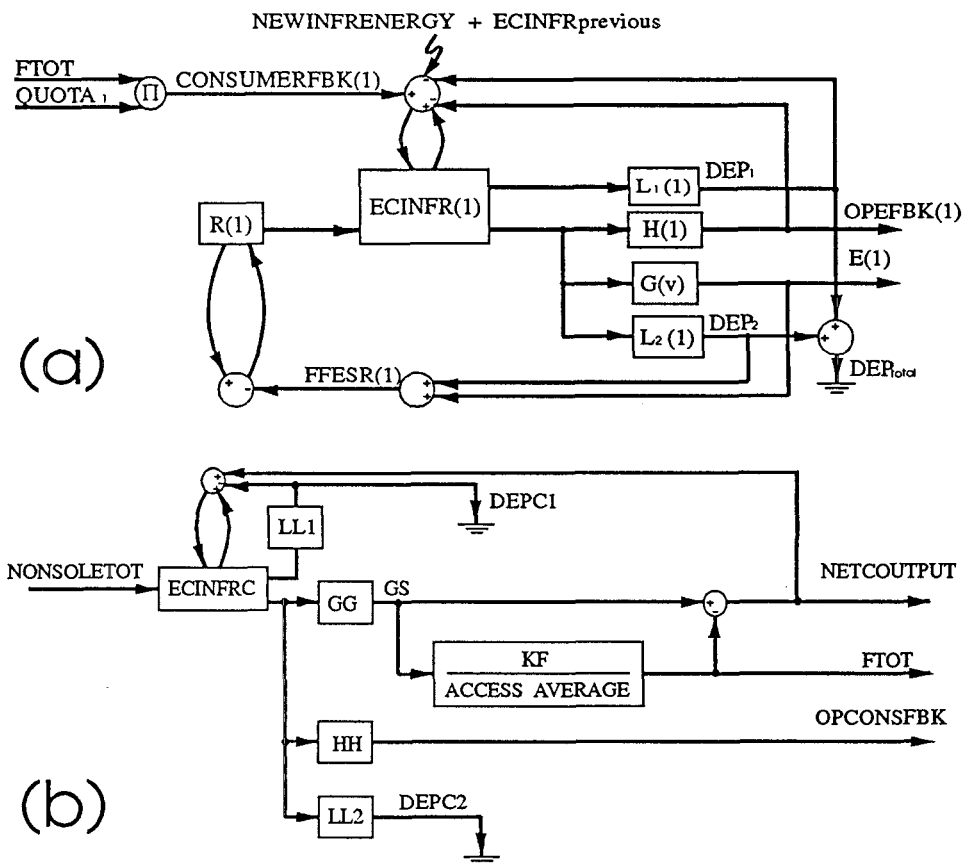


Figure 5.7. Flow diagrams for the improved equation in (a) Energy Supply Sectors, and (b) the Socio-Economic Sector.

of the previous section would adequately account for waste.

The additional waste $DEP_2(V)$ in equation 5.7 was at first assumed to effect the EEDSM equation (5) for NEWINFREENERGY - new Energy Sector infrastructure

$$NEWINFREENERGY(v) = CONSUMERFBK(v) - OPEFBK(v) - DEP(v) \quad \text{EEDSM(5)}$$

as follows

$$NEWINFREENERGY(v) = CONSUMERFBK(v) - OPEFBK(v) - DEP_1(v) - DEP_2(v) \quad (5.18)$$

However the reasoning that led to equation 5.17 showed this assumption to be wrong.

Thus the 5.18 becomes

$$NEWINFREENERGY(v) = CONSUMERFBK(v) - OPEFBK(v) - DEP_1(v) \quad (5.19)$$

This equation in turn effects equation 5.8 for $ECINFR_{j+1}(V)$. With full substitution of EEDSM(21), (1), 5.6, and 5.19, the equation 5.8 becomes

$$ECINFR_{j+1}(v) = ECINFR_j(v) * (1 - L_1(v) - H(v) * R(v)) + (FTOT * QUOTA_v) \quad (5.20)$$

new infrastructure(V) =

$$\text{infrastructure}(V) * (1 - c10 - c11 * \text{Resource size}) + (\text{Soc-Eco feedback} * \text{Quota}_v)$$

where $c10$ is the coefficient in the equation for $DEP_1(V)$, $c11$ in the equation for $OPEFBK(V)$, and V indicates a specific energy resource.

The waste in energy refining and transportation $DEP_2(V)$ has an effect on the EEDSM equation (14). Equations 5.7 and 5.17 are substituted into EEDSM(14), which becomes

$$R_{j+1} = R_j(v) * (1 - ECINFRC_j(v) * (L_2(v) + G(v))) \quad (5.21)$$

new Resource size(V) =

$$\text{Resource size}(V) * (1 - \text{infrastructure} * (c12 + c13))$$

where c12 is a coefficient in the equation for $DEP_2(V)$, and c13 is the equation for $E(V)$.

These equations are drawn as a flow diagram in Figure 5.5(a).

There were no such alterations indicated in the flow diagram for the Energy Supply Sector, Figure 5.4(b).

To summarise, the alterations to equations affected three of the four equations chosen for investigation. Those investigated in Section 5.3 are equations 5.16, EEDSM(13), 5.20 and 5.21.

5.3 Investigating the non-linear equations

The investigation of an equation involved interpreting the behaviour of that equation. Initially each equation was carefully examined. Each component involved in the equation was investigated to find its purpose and to understand how it might affect the equation. Possible coefficient value ranges were considered. The aim here was to determine critical values as well as ranges that would lead to physically impossible results.

Each equation was then programmed in MathLab's programming language as shown in Appendix B. The programs were designed to vary one or two coefficients or components of the equation. This allowed changes in behaviour relative to changing values to be seen in the resulting plots.

We will now discuss the equations individually.

Equation 5.16 (which is repeated below) has three coefficients GG, KF and LL_1 . As previously stated, these values were to be estimated and fixed for the investigation.

$$ECINFRC_{j+1} = ECINFRC_j * (1 - LL_1 + NONSOLETOT * (GG * (1 - KF/ACCESSAVERAGE))) \quad (5.16)$$

GG is the coefficient that specifies the amount of energy

that enters into Goods & Services production. Using the relationship in equation 5.11 (with $KK = 1$) GG could be estimated from the other coefficients.

An estimate was made of the efficiency of the major energy forms coal, petrol, diesel and electricity. The result was an estimate of 40% energy waste during conversion to electricity - $DEPC_2$.

Combining this with a estimated feedback to the Socio-Economic Sector's infrastructure of 10% of the useful energy, leaves a figure of 0.54 for GG.

KF is the coefficient in the equation calculating FTOT - the proportion of Goods & Services which are feedback to the Energy Supply Sectors. KF must be a positive number, since the other variables and the product of EEDSM(18) are positive. Also FTOT is a proportion of GS (as just mentioned), hence

$$0 \leq \frac{KF}{ACCESSAVERAGE} \leq 1 \quad (5.22)$$

In a business, the amount spent on energy would be one of the five major expenses - including labour, raw materials, plant, and profit paid to investors/creditors. Hence energy would be, on average 20% of a businesses' spending. This figure was used for the value of $KF/ACCESSAVERAGE$, hence FTOT was set to 20% of Goods & Services produced.

The equation 5.16 with substituted coefficients values is

$$ECINFRC_{j+1} = ECINFRC_j * (1 - 0.2 + NONSOLETOT * (0.54 * (1 - 0.2))) \quad (5.16)$$

which simplifies to

$$ECINFRC_{j+1} = ECINFRC_j * (0.8 + 0.432 * NONSOLETOT) \quad (5.22)$$

The investigation examined two aspects. The first was the objective stated in the previous section, finding how $ECINFRC_{j+1}$ would change with changing NONSOLETOT. There was also an interest in behaviour for a variation of the coefficients along their ranges, 0 to 1.

The results of the behaviour with changing NONSOLETOT are shown in Figure 5.6(a). The equation adds a positive values to the previous ECINFRC size for any value of NONSOLETOT over the critical value calculated in equation 5.23

$$\text{Critical Value} = \frac{LL_1}{GG * (1 - KF/ACCESSAVERAGE)} \quad (5.23)$$

which in this case was ≈ 0.463 .

This causes successive iterations to quickly increase ECINFRC towards positive infinity.

The figure 5.6(a) shows values of NONSOLETOT of between 0 and 4. A normal value for NONSOLETOT was 1000 [Bodger and May, EEDSM fortran program, unpublished]. If this value of NONSOLETOT were used with an initial value for ECINFRC_j of 1, and the equation was allowed to run for five iterations (the conditions that generated Figure 5.6(a)), ECINFRC_{j+1} would be approximately $1.5 * 10^{13}$.

For the examination of the sum of coefficients GG and KK/ACCESSAVERAGE (from here on called "SoC"), LL₁ was again set to 20% and NONSOLETOT was set to 1000.

Figure 5.6(b) shows the results for the 0 to 1 range of SoC. SoC has an effect on the increase of ECINFRC_{j+1}, but would not make any real impression unless its value was less than 0.1. This would imply either low efficiency, high feedback to the infrastructure, or high feedback to the Energy Supply Sectors. All of these changes would be bad for the viability of any business.

On an individual basis, KF/ACCESSAVERAGE could be increased above 1, but this would mean that all the FTOT was being fed back to the Energy Supply Sectors, as well as energy stored in the infrastructure. This possibility was not considered in the EEDSM description.

Figure 5.6(c) shows the impact of the DEPC₁ to be very small on the outcome of the equation. ECINFRC_{j+1} was increasing too fast for it to have any real effect.

It appeared that EEDSM equation (13)

$$NONSOLETOT_{j+1} = NONSOLETOT_j = COMM - COMMO \quad \text{EEDSM(13)}$$

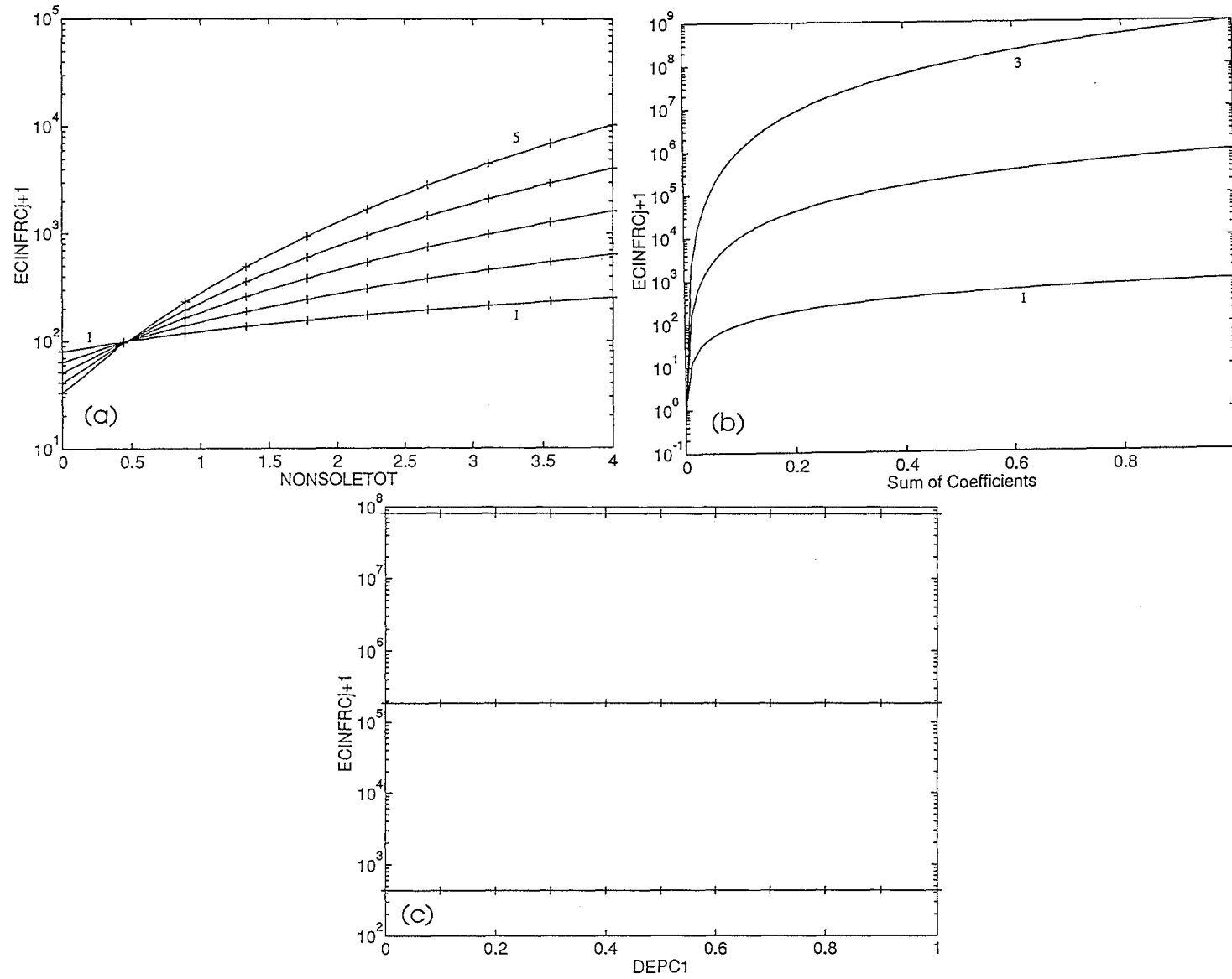


Figure 5.8. $ECINFR_{j+1}$ values for changing (a) NONSOLETOT, (b) sum of coefficients GG, and KF/ACCESSAVERAGE, and (c) $DEPC_1$. The lines indicate successive iterations.

would be difficult to examine, because COMMO was determined by the previous NONSOLETOT. The route between the two was EEDSM(17), (18), (21), (4), (2): for each Energy Supply Sector) and (7a).

However this problem was solved through forming several assumptions. (1) That NONSOLETOT was the total amount of energy offered to the Socio-Economic Sector, (2) a part of which became GS, (3) part of GS became FTOT, and (4) part of FTOT was divided up into the CONSUMERFBK's. Assumption (5) was that CONSUMERFBK(V) provided some change in the output of each R(V) - hence COMMO.

Assumptions 1 through 3 suggested that CONSUMERFBK(V) was a proportion of NONSOLETOT, which could be represented by a simple relationship such as

$$\text{CONSUMERFBK}(v) = \text{multiplier1} * \text{NONSOLETOT} \quad (5.24)$$

where the multiplier had a range between 0 and 1.

However the effect of a change in CONSUMERFBK had a less obvious link to a change in COMMO. Varying CONSUMERFBK would change the size of a Energy Supply Sector's infrastructure, which had a direct bearing on the ability of the sector to process energy. The size of the effect though, was difficult to define.

An approximation was to use a multiplier2, with a range between 0 and 2 to account for an increase or decrease in E(V), and extend equation 5.24 to a relationship between the sum of the E(V)'s and NONSOLETOT.

$$\text{COMMO} = \text{multiplier2} * \text{NONSOLETOT} \quad (5.25)$$

where multiplier3 is the product of the two previous multipliers and so has a range of 0 to 2.

Thus the equation which was examined was EEDSM(13) with 5.25 substituted into it.

$$\begin{aligned} \text{NONSOLETOT}_{j+1} = & \text{NONSOLETOT}_j \\ & + \text{M3} * (\text{NONSOLETOT}_{j-1} - \text{NONSOLETOT}_j) \end{aligned} \quad (5.26)$$

where m3 is multiplier3.

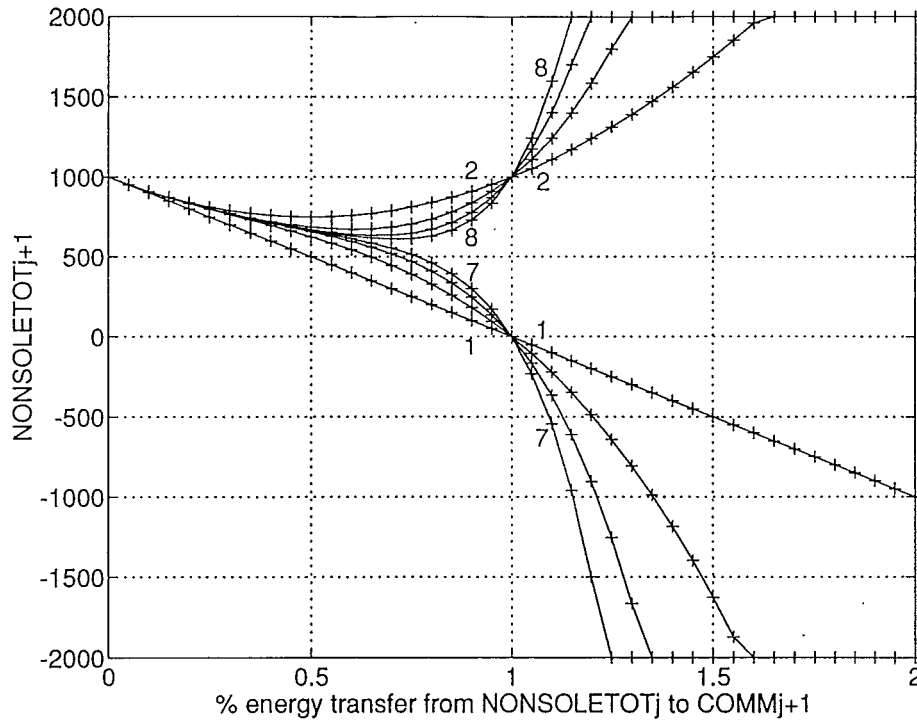


Figure 5.9. NONSOLETOT_{j+1} values for a changing ratio of COMM_{j+1} derived from NONSOLETOT_j . The lines indicate successive iterations.

The result of varying multiplier m_3 is seen in Figure 5.7. For a value of $0 \leq m_3 \leq 1$ the iterations converge towards a central value. At $m_3 = 1$, the behaviour steps continuously between two values of NONSOLETOT , the initial value and zero (in this case an initial value of 1000 was used).

For values m_3 where $1 \leq m_3 \leq 2$, the behaviour diverges, alternating between positive and negative numbers on route to both infinities.

It was not possible however for there to be a negative store of energy, so any negative result for NONSOLETOT was incorrect. This showed that m_3 must be maintained below a maximum value of 1 and a minimum of 0. Only the behaviour between 0 and 1 was significant.

The equation 5.19 for ECINFR_{j+1}

$$\text{ECINFR}_{j+1}(v) = \text{ECINFR}_j(v) * (1 - L_1(v) - H(v) * R(v)) + (\text{FTOT} * \text{QUOTA}_v) \quad (5.19)$$

had four variables, and each had ranges that were estimable.

CONSUMERFBK(V) is the proportion of FTOT (feedback to Energy Supply Sectors) allocated to a particular energy type. The amount of CONSUMERFBK is relative to that sector's market share.

Although the value of CONSUMERFBK would on average be a fifth of the FTOT (which was approximately an fifth of NONSOLETOT), the maximum of the range was set to the value used for NONSOLETOT (in this case 1000) the same as for the previous equation. $L_1(V)$ is the coefficient that specifies the proportion of ECINFR(V) which decays in the passage of an iteration. This suggested that a range of 0 to 1 was appropriate.

H(V) is the coefficient used to calculate OPEFBK(V) the amount of energy stored in ECINFR(V), which was required to be consumed each iteration. It was also given a range of 0 to 1.

R(V) is the size of the energy resource of a Energy Supply Sector. It is a value that decreases as energy is removed from it - R(V) decreases each iteration. In the examination of equation 5.19's behaviour the author set the depletion of R(V) as if it would be totally consumed over 20 years. However the constant proportion of 1/20 was removed from the total remaining, not 1/20 of the initial resource. This impresses the point that the amount of remaining resource has a bearing on the exploitation of it.

The values for R(V) used for the EEDSM range from 1000 to 36000. For this examination R(V) was set to range between 0 and 40000.

As equation 5.19 was tested, only one variable at a time was varied (except of course, R(V)). The other variables were set a coefficient value as follows

CONSUMERFBK(V):- a fifth of a fifth of 1000 (NONSOLETOT), 40.

$L_1(V)$:- 20% decay, 0.2.

H(V):- 10% of 1/20th of R(V) (at 90% efficiency), 0.045.

R(V):- 10000.

With these values substituted in, equation 5.19 becomes

$$ECINFR_j(v) = ECINFR_j(v) * (1 - 0.2 - 0.045 * 10000) + 40 \quad (5.27)$$

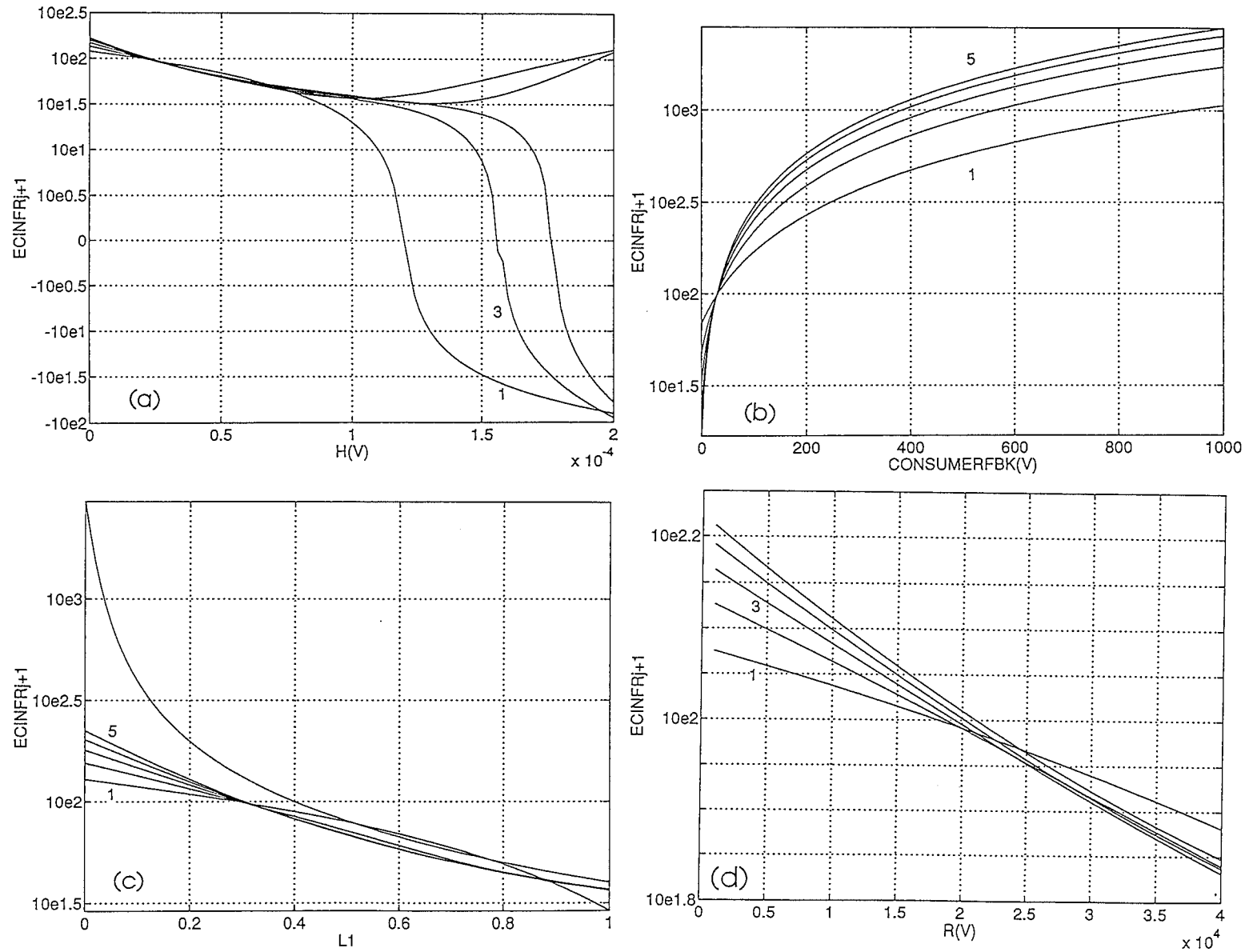


Figure 5.10. $ECINFR_{j+1}(v)$ values for changing (a) $H(v)$, (b) $CONSUMERFBK(v)$, (c) $L_1(v)$, and (d) $R(v)$. The lines indicate successive iteration (numbered).

Whilst examining this equation with varying $H(v)$, it was discovered that $H(v)$ was set too high. For values of $H(v)$ greater than $2 \cdot 10^{-4}$, the iteration values for $ECINFR_{j+1}$ would diverge towards negative and positive infinity, with all odd numbered iterations being negative numbers.

This is an extremely unstable way to run the system - it simply would not work.

The behaviour for $H(v)$ less than $2 \cdot 10^{-4}$ is shown in Figure 5.10(a). All of these values result in convergence to a positive value of $ECINFR_{j+1}(v)$. For values below $1.2 \cdot 10^{-4}$, the odd numbered iterations return positive values, which converge faster with the even numbered iterations. This region is where any healthy business would be operated.

It was considered that the other plots that had been generated, might be artefacts of an incorrect selection of $H(v)$. The work repeated with the constant value of $H(v)$ set to $1.1 \cdot 10^{-5}$. Equation 5.27 becomes

$$ECINFR_{j+1}(v) = ECINFR_j(v) * (1 - 0.2 - 1.1 \cdot 10^{-5} * 10000) + 40 \quad (5.28)$$

The previous plots were quickly confirmed as being incorrect.

Figure 5.10(b) shows the behaviour for varying $CONSUMERFBK(v)$. A critical point in the plot is calculated by

$$CV_{CONSUMERFBK(v)} = ECINFR_j(v) * (L_1(v) + H(v) * R_j(v)) \quad (5.29)$$

If $CONSUMERFBK(v)$ is set to this value and the other coefficients remain set, $ECINFR_{j+1}(v)$ will remain constant. A small increase/decrease in $CONSUMERFBK$ will give a large continuous increase/decrease for as long as $CONSUMERFBK$ remains non-critical.

equation to converge. The sixth line on the plot shows iteration number 100.

Figure 5.10(d) shows the equation behaviour for varying $R(v)$. There is also a critical value of $R(v)$ calculated by

$$CV_{R(v)} = \frac{(CONSUMERFBK(v)/ECINFR_i(v)) - L_1(v)}{H(v)} \quad (5.30)$$

In this case the value was 18181.81, where a lower/higher value results in $ECINFR_{j+1}(v)$ converging to a higher/lower figure. All values of $R(v)$ in this range converge, with $R(v) = 0$ so that initial values of $ECINFR_{j+1}(v) = 200$ and $R(v) = 40000$ give an $ECINFR_{j+1}(v) = 62.5$.

The equation for $R_{j+1}(V)$, 5.20, showed how the total flow of energy from the resource becomes $G(V)$ - the amount of energy that was successfully transferred from the resource to the Energy Storage Sector, and $L_2(V)$ - the proportion of energy that became waste in the process.

The ranges that these coefficients may have, was not an issue, as they summed together to give the coefficient $K(V)$, from EEDSM equation (3)

$$FFESR(v) = K(v) * ECINFR(v) * R(v) \quad \text{EEDSM(3)}$$

This meant that the examination may as well have been of equation 5.8

$$R_{j+1}(v) = R_j(v) * (1 - K(v) * ECINFR_j(v)) \quad (5.8)$$

as the size of $K(V)$ was the only factor that effected $R(V)$. With $R(V)$, efficiency was not an issue. This resulted in a range of $0 \leq K(V) \leq 2$ being selected, as a first estimate.

The values for $ECINFR_j(V)$ varied widely in the runs of Bodger and May's version of the EEDSM. A full range of 0 to 20000+ was indicated. The full extent of this range was not examined, a decision made in ignorance which proved to be worth while. A range of 0 to 1000 was used.

Figure 5.11(a) shows the values of $R_{j+1}(V)$ generated with varying values of $K(V)$ ($ECINFR_j(V)$ was set to 100).

Five iterations are shown, the results alternating between negative and positive values as the numbers got rapidly larger, on their way to infinity. However it was not possible to have a negative resource size -

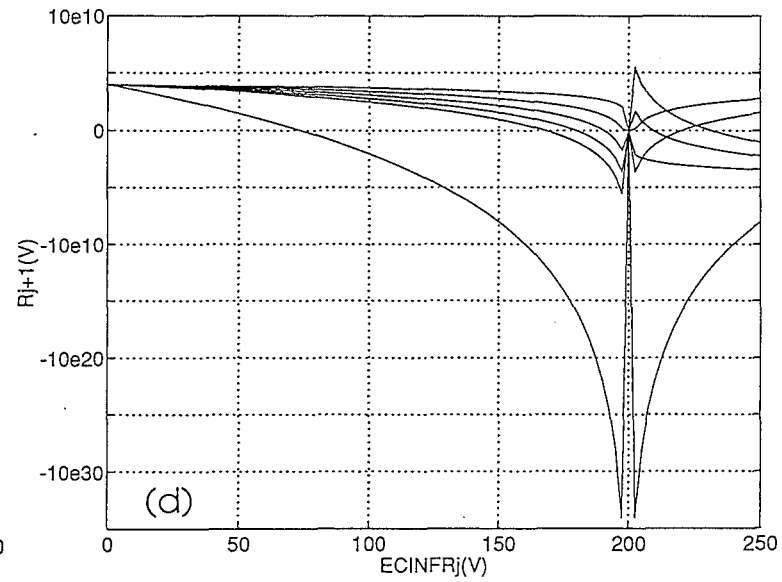
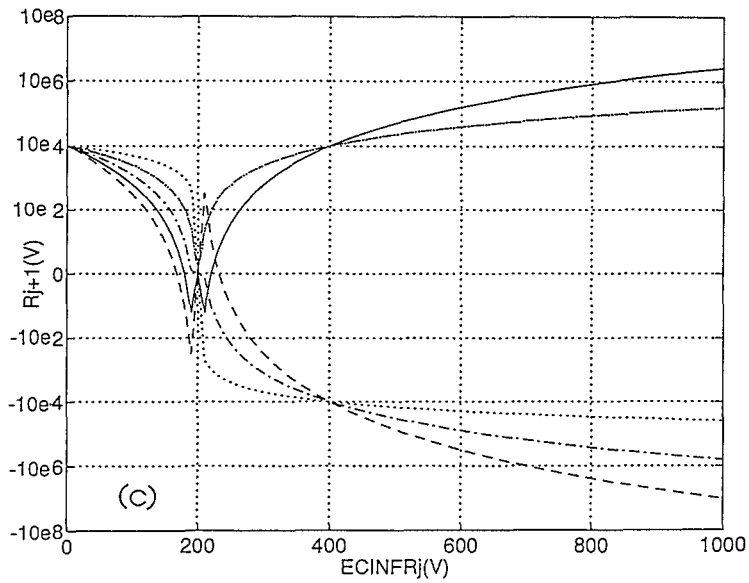
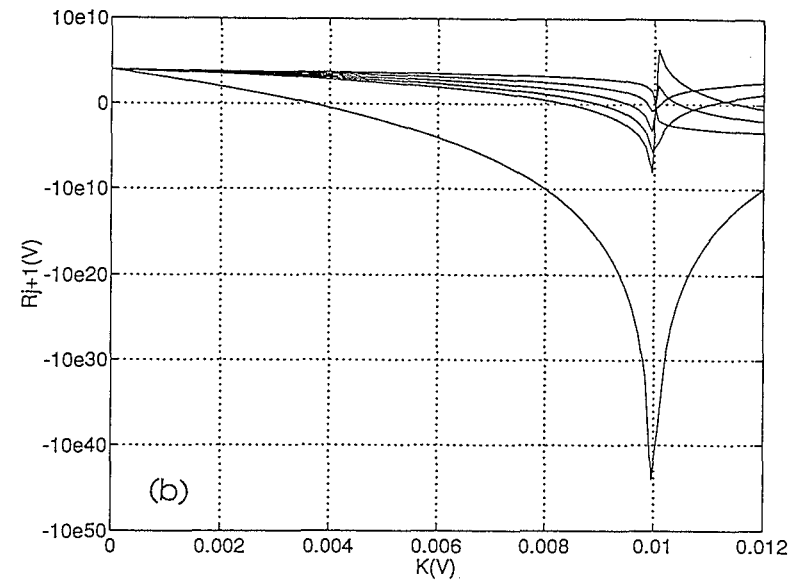
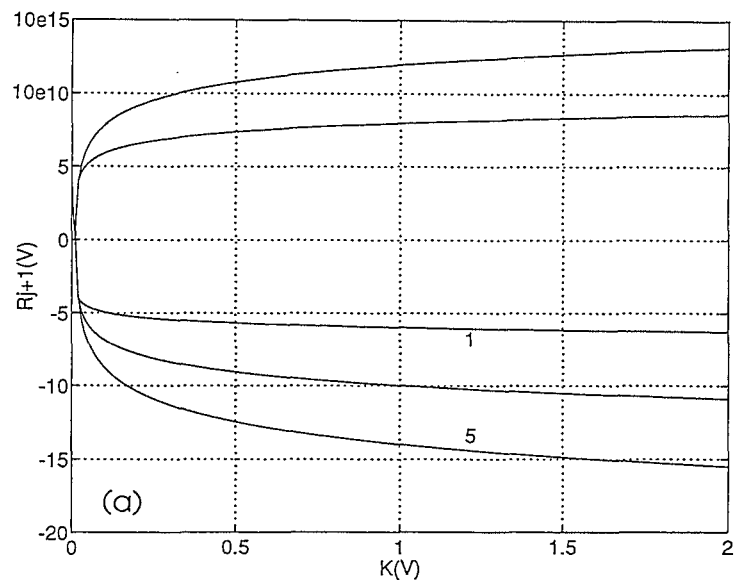


Figure 5.11. Values of $R_{j+1}(v)$ for varying coefficients, (a) $K(v)$, and (c) $ECINFR_j(v)$. The plots (b) and (d) are for small values of $K(v)$ and $ECINFR_j(v)$.

most values of $K(V)$ appeared to give nonsensical results.

A close look at the far left of (a) shows that there may be different behaviour there. Figure 5.11(b) proves this to be correct. For values of $K(V)$ below 0.01, successive iterations were initially positive, decreasing $R_j(V)$ towards zero (between 0.008 and 1, only five iterations would bring $R_j(V)$ to zero). The negative values are nonsense values as before. These curves helped the author to decide on a coefficient value of 0.005 for $K(V)$ whilst varying $ECINFR_j(V)$.

The plot in Figure 5.11(c) shows that even the range of 0 to 1000 was too large. Negative values for the first iteration occur after $ECINFR_j(V) = 200$.

Figure 5.11(d) is an enlargement of (c) showing the behaviour below the critical point. The critical point is obvious if one examines equation 5.8. When

$$\begin{aligned} ECINFR_j(v) * K(v) &= 1 \\ \rightarrow R_{j+1}(v) &= 0 \end{aligned} \tag{5.31}$$

and any further increase in either coefficient, would result in a negative $R_{j+1}(V)$ value.

5.4 Discussion

The equations examined are non-linear, however this in not enough in itself for the equations to exhibit chaotic behaviour. As a measure of the existence of chaos the bifurcation (or not) of the equation behaviour is used to determine if the equation is sufficiently non-linear. However non-linearity does not have a magnitude over which chaotic behaviour will occur. It is the structure of the equations of the system itself that enable chaos to exist. If the coefficients of the equations are also over certain values of magnitude then chaotic behaviour will occur.

There was no evidence of bifurcation occurring in any of the equations examined. As was stated in section 2.3 there appears to be a minimum requirement for a square term in an equation for that equation to exhibit any chaotic behaviour. Examining the equations shows that there are no squared terms and that no chaotic behaviour exists. The

hypothesis would appear to be correct. Hence it is extremely unlikely that the EEDSM equations are chaotic.

The testing of the equations helped to redefine several of the equation coefficient ranges. In determining the new infrastructure size in the Socio-Economic Sector $ECINFRC_{j+1}$, the author described a critical value (calculated in equation 5.23) for $NONSOLETOT_j$. If $NONSOLETOT$ could be maintained around this value, $ECINFRC_{j+1}$ could be increased slightly by an increase in $NONSOLETOT$ or decreased by a reduction in $NONSOLETOT$.

The equation in question is equation 5.16:

$$ECINFRC_{j+1} = ECINFRC_j * (1 - LL_1 - NONSOLETOT * (GG * (1 - KF/ACCESSAVERAGE))) \quad (5.16)$$

As was mentioned in section 5.3 the critical $NONSOLETOT$ value was calculated as being only 0.04% of the value used by Bodger and Hayes.

This problem can be overcome by reducing the size of the coefficient KK from the ideal value of 1. A factor of ten decrease would increase the critical $NONSOLETOT$ value by the same amount.

This decrease of KK would not reduce the size of outputs such as $FTOT$, since $ECINFRC$ would be offered a larger valued $NONSOLETOT$. However $ECINFRC$ would not be able to access as much of the offered energy as before. $ECINFRC$ would need to be enlarged to transfer the same amount of energy.

The next examination was of equation 5.26 which calculates the next quantity of energy offered by the Energy Storage Sector, $NONSOLETOT$.

$$NONSOLETOT_{j+1} = NONSOLETOT_j + M3 * NONSOLETOT_{j-1} - M3 * NONSOLETOT_j \quad (5.26)$$

Recall that the behaviour is confined by the impossibility of having a negative quantity of energy. However this means that the values for

NONSOLETOT_{j+1} can never be larger than the initial value, though the NONSOLETOT_{j+1} will converge to a value, providing the proportion of COMM_{j+1} relative to NONSOLETOT_j (m_3 - see equation 5.26) remains coefficient.

This not only implies that NONSOLETOT cannot be enlarged, but also that any change in this $\text{COMM}_{j+1}/\text{NONSOLETOT}_j$ ratio would produce a smaller NONSOLETOT_{j+1} . Eventually this must reduce NONSOLETOT to zero.

If there is no way of increasing NONSOLETOT , then a way should be built into the equation. This change should also enable the reduction of NONSOLETOT without needing to adjust the $\text{COMM}_{j+1}/\text{NONSOLETOT}_j$ ratio (which would mean less efficiency in the energy flow from NONSOLETOT_j to COMM_{j+1}).

Adding a coefficient to EEDSM equation (13) will allow adjustment where necessary. The equation is that below

$$\begin{aligned} \text{NONSOLETOT}_{j+1} \\ = \text{NONSOLETOT}_j + \text{COMM} - \text{COMM}_0 + \text{ADJUSTVALUE} \end{aligned} \quad (5.32)$$

where Adjustvalue would be set to zero except when it was being used to adjust the value of NONSOLETOT .

The examination of equation 5.20 for ECINFR_{j+1}

$$\text{ECINFR}_{j+1}(v) = \text{ECINFR}(v) * (1 - L_1(v) - H(v) * R(v)) + (\text{FTOT} * \text{QUOTA}_v) \quad (5.20)$$

showed that one must be careful in selecting coefficients. The first choice of $H(V)$ caused the equation to operate in a region that produced unacceptable results.

It is not enough to simply let equations run and report the results, the results must be fully interpreted to ensure that the behaviour is possible. The computer does not know that a particular variable cannot have a negative value.

The important coefficient for this equation is $H(V)$. It must remain in a range of 0 to 2×10^{-4} to give sensible converging results.

$\text{CONSUMERFBK}(V)$ has a critical value that can be used to

constrain the equation, keeping $ECINFR_{j+1}$ constant, increasing , or decreasing it. This is the way that the Socio-Economic sector (or even Energy Supply Sector) can control the production of energy.

Choice of $L_1(V)$ and $R(V)$ only affects the ability of the equation to converge. But there is a danger in that an improvement (reduction of $L_1(V)$) in the decay of the infrastructure increases the number of iterations required for convergence. Can efficiency produce instability? This is a frightening prospect!

The theme of correct coefficient choice was repeated for equation 5.9

$$R_{j+1}(v) = R_j(v) * (1 - (K(v) * ECINFR_j(v))) \quad (5.9)$$

for $R_{j+1}(V)$, the new resource size. The range of $K(V)$ which produces useful results is 0 to 0.01. In this range $R_{j+1}(V)$ decreases as each iteration leads to removal of a proportion of the resource.

The plots in Figure 5.11(b) and (d) have a sixth line added to them, the curves at the bottom of the plots. These represent the twentieth iteration, or the twentieth year of resource depletion. If the resource was to be used over twenty years, then the coordinate values relating to these lines crossing $R_{j+1}(V) = 0$ should be selected as coefficient values. In this case for $R_{initial}(V) = 10\ 000$, the coordinates would be $K(V) = 3.92 \times 10^{-3}$ and $ECINFR_j(V) = 75$.

These plots could be repeated for any resource size to help determine desirable coefficient values and lengths of exploitation time.

This is exactly what is needed to enable the EEDSM programme to be re-written. The author did enter the adapted equations into the programme, however this in itself did not allow for computation of the system. The initial values used by Bodger and May are not suitable for the adapted equations. The result was that the values calculated in the programme quickly converged to positive infinity. Thus the programme ended in error before completion of the problem.

The work on coefficients which has just been discussed will aid in calculation of initial values which will enable the adapted EEDSM programme to be operated and further examined. However that is a task for a future research student.

6 Conclusions

There are two major conclusions to draw from this research. The first is that the altered EEDSM equations do not have any squared terms and do not exhibit any bifurcation behaviour and hence are unlikely to be chaotic. It is assumed that this result is the same for the original EEDSM.

The second conclusion is that estimates of coefficient ranges now exist. The examination of the altered equations indicated that several coefficients are required to have their values within specific ranges, for the results to be meaningful. These ranges are given in section 5.4.

This information will be of use in programming and running the altered EEDSM. Coefficient values enable the system to match trends in energy consumption. The method of finding these values would appear to be one of trial and error. The ranges suggested in this thesis will greatly reduce the work required in this endeavour.

The logistic equation has been used to model energy consumption [Marchetti 1977, 1979, 1980]. This equation has a squared term, and its behaviour can be driven to bifurcate. Further, various authors have stated that the logistic equation exhibits chaotic behaviour [Devaney 1989].

The question arises "does real world energy consumption exhibit chaotic behaviour?" Would a model that estimates this process need to be chaotic, or not chaotic, to match the process?

It would be an interesting exercise to use Wolf's Lyapunov exponent determining methods on actual energy consumption data. A years worth of New Zealand half hourly electricity consumption data was available. However the author did not have a chance to make use of it.

The data could be insufficient to determine long term trends. Short term trends could interfere with estimation, making modelling difficult. Such difficulties could be overcome if additional data became available. Ten years worth of data might prove sufficient.

Appendix A - Software Packages

A.1 Dstool - A Dynamic System Toolkit with an Interactive Graphical Interface

Dstool is a equation-behaviour visualisation package. An equation is computed analytically in multiple dimensions. The coordinates are viewed in two dimensions, in multiple viewer selected windows.

The package is Unix window based run of Sun Sparc Micro-stations, utilising a mouse, pop-up menus and application windows which enable numerical input, complex manipulation of initial conditions and tracking of system behaviour as it develops.

The software accommodates Dynamic Systems that are mappings (iterative equations) or vector fields (partial derivative equations). Supplied documentation claims that these two methods are generally the only way of obtaining trajectory information.

The interactive qualities of the package are excellent due to the often intricate geometric structures of a dynamic system as well as their sensitivity to changes in parameters. Interaction is through mouse selection of coordinates or through application windows.

Dstool is based on a program "Kaos", this title suggests the use for both programs, to visualise chaotic behaviour. Indeed the phase space view of behaviour along with detection of critical points (sources, sinks and saddle points) and in observation of affects on behaviour through coordinate changes.

In Figure A.1(a),(b) an example of such a change can be seen affecting the Lorenz attractor.

Although the author enjoyed operating Dstool, and learnt a great deal about chaos through using it, he quickly became frustrated with the package. Introducing a new dynamic system to the program was unnecessarily complicated, and very limiting. As an example, the program requires one to enter the plotting information with the equation - one has to know what they want to see before you have seen it.

It was this type of inflexibility that lead to a change to Matlab.

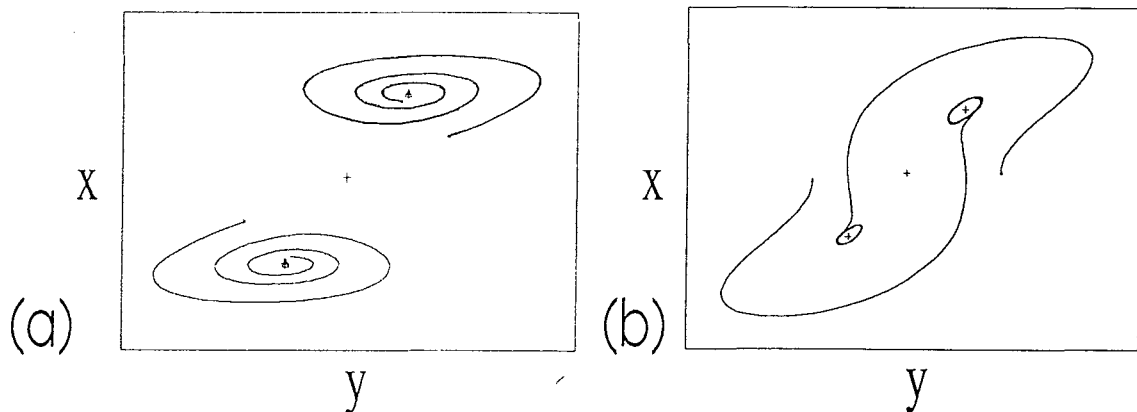


Figure A.1. The Lorenz attractor for two different values of coefficient s .
 (a) $s = 2$, behaviour of two spiral sinks and a central saddle point,
 (b) $s = 10$, three saddle points.

A.2 Matlab - Matrix Laboratory

Matlab is an interactive, matrix based system for scientific and engineering calculation. It is licensed software from The MathWorks Inc., and was operated on a Sun Sparc Micro station.

The user is provided with a high level language which enables data to be created (loaded), plotted and printed with ease.

A program was written to examine the behaviour of an equation across a range of coefficients. If the equation is one that shows bifurcation behaviour this will be seen on the resulting plot.

The program was adapted to take advantage of Matlab commands which allow inputs for successive runs of the program, to be selected from the plot. This enabled the user to begin with a wide range of coefficient values (a wide view of behaviour), note the areas of interest on the plot, and recalculate for that area (zoom in on interesting behaviour).

This program was immensely useful while examining equations. The areas of interest were quickly located, allowing initial conditions to be selected for the "presentation plots" (as in Figure 4.1).

A.3 Fortran - Formula Translator 1977

Fortran-77 is a computer language commonly used for engineering applications. This is the language that the previous version of the EEDSM was written, although it was compiled on the Electrical Engineering Departments DEC Vax. The text file containing the program was transferred to a Sun Sparc Micro station, and compiled there.

The improvements to the EEDSM that are discussed in Section 5.2 were programmed and compiled in f77. Further comment appears in Section 5.4.

Appendix B - Equations of the original EEDSM

- 1 $OPEFBK(V) = H(V) * ECINFR(V) * R(V)$
- 2 $E(V) = G(V) * ECINFR(V) * R(V)$
- 3 $FFESR(V) = K(V) * ECINFR(V) * R(V)$
- 4 $ECINFR_{j+1}(V) = ECINFR_j(V) + NEWINFREENERGY(V)$
- 5 $NEWINFREENERGY(V) = CONSUMERFBK(V) - OPEFBK(V) - DEP(V)$
- 6 $DEP(V) = L(V) * ECINFR(V)$
- 7a $COMM0 = \Sigma E(V)$
- 7b $NONSOLARSHARE(V) = E(V) / COMM0$
- 8 $FO(V) = NONSOLARSHARE(V) / (1 - NONSOLARSHARE(V))$
- 9 $ACCESS(V) = E(V) / OPEFBK(V)$
- 10 $ACCESSAVERAGE = \Sigma (ACCESS(V) * NONSOLARSHARE(V))$
- 11 $QUOTA(V) = ACCESS(V) * NONSOLARSHARE(V) / ACCESSAVERAGE$
- 12 $NEWINFRTOT(V) = \Sigma NEWINFREENERGY(V)$
- 13 $NONSOLETOT_{j+1} = NONSOLETOT_j + COMM - COMM0$
- 14 $R(V)_{j+1} = R(V)_j - FFESR(V)$
- 15 $OPCONSFNK = HH * NONSOLETOT * ECINFRC$
- 16 $DEPC = LL * ECINFRC$
- 17 $GS = GG * NONSOLETOT * ECINFRC$
- 18 $FTOT = KF * GS / ACCESSAVERAGE$
- 19 $NETCOUTPUT = GS - FTOT$
- 20 $NEWINFRCONSUMER = NETCOUTPUT - OPCONSFBK - DEPC$
- 21 $CONSUMERFBK(V) = QUOTA(V) * FTOT$
- 22 $ECINFRC_{j+1} = ECINFRC_j + NEWINFRCONSUMER$

There are seven more equations, each of which is one of the above equations re-written with respect to the coefficient. Hence there are equations for HH, LL, GG, K(V), H(V), L(V), and G(V). There is no equation for KF.

Appendix C - Matlab programs

The following are the Matlab programs written for the research task. Families of programs are given together with only their differences shown.

All programs are based around a program written by Dr. David Wall.

1 Logistic equation

```
% A routine to plot the bifurcation diagram
% for a real valued function
% Initialisation
xmin=-0.5;pin=-1.7;xmax=1;pf=4;
xi=0.05;ni=80;nt=200;np=300;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=parm*x*(1-x); % change this definition if different function
        if i> ni
            if x > xmax, x=xmax;
            elseif x < xmin, x=xmin;
            end
            z(i-ni,j)=x;
        end
    end;
end
% Plot and print commands followed
```

2 Simple feedback equation - for c and β .

```
% A routine to plot the bifurcation diagram
% for a Simple feedback equation
%  $y = (c - \beta * x)^2$  with varying c = parm, 0 -> 2.
% Initialisation
xmin=0;pin=0;xmax=4;pf=2;
xi=0;ni=50;nt=100;np=150;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=(parm-x)^2; % change this definition if different function
        if i> ni
            if x > xmax, x=xmax;
            elseif x < xmin, x=xmin;
            end
        end
    end
end
```

```

        end
        z(i-ni,j)=x;
    end
end;
end
% Plot and print commands followed
% END

% A routine to plot the bifurcation diagram
% for a Simple feedback equation
%  $y = (c - \beta * x)^2$  with varying  $\beta = \text{parm}$ ,  $0 \rightarrow 2$ .
% Initialisation
xmin=0;pin=0;xmax=4;pf=2;
xi=0;ni=50;nt=100;np=150;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=(1-parm*x)^2; % change this definition if different function
        if i > ni
            if x > xmax, x=xmax;
            elseif x < xmin, x=xmin;
            end
            z(i-ni,j)=x;
        end
    end
end;
end
% Plot and print commands followed
% END

```

3 ECINFRCj+1 equations, ecl.m, ecn.m and ecs.m.

```

% A routine to plot the bifurcation diagram
% for a real valued function
% in this case ECINFRC = x,
% with varying NONSOLETOT = constant = 1000,
% Summed coefficients which could be  $0 \rightarrow 1$ , set as 0.432,
% and L1 which could be  $0 \rightarrow 1$ ,  $0 \leq \text{parm} \leq 1$ .
xmin=-20000; xmax=2000000000; pin=0; pf=1;
nt=3;np=11; xi=1;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);k=zeros(3,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=x*(1+1000*0.432-parm); % change this definition if diff funct
        if x > xmax, x=xmax;
        elseif x < xmin, x=xmin;
        end
        z(i-ni,j)=x;
    end
end

```

```

        if i<3
            k(i+1,j)=x;
        end
    end;
end
% Plot and print commands followed
% END

% A routine to plot the bifurcation diagram
% for a real valued function
% in this case ECINFRC = x,
% with varying NONSOLETOT = parm, 0 -> 4,
% Summed coefficients which could be 0->1, set as 0.432,
% and L1 which can be 0->1, set as 0.2.
xmin=-20000; xmax=20000; pin=0; pf=4;
nt=5; np=10; xi=100;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=x*(1+0.432*parm-0.2); % change this if diff funct
        if x > xmax, x=xmax;
        elseif x < xmin, x=xmin;
        end
        z(i-ni,j)=x;
    end;
end
% Plot and print commands followed
% END

% A routine to plot the bifurcation diagram
% for a real valued function
% in this case ECINFRC = x,
% with varying NONSOLETOT = constant =1000,
% Summed coefficients = parm, 0 <= parm <= 1,
% and L1 which could be 0->1, set as 0.2.
xmin=-20000; xmax=2000000000;pin=0; pf=1;
nt=3; np=81; xi=1;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);k=zeros(3,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=x*(1+1000*parm-0.2); % change this if diff funct
        if x > xmax, x=xmax;
        elseif x < xmin, x=xmin;
        end
        z(i-ni,j)=x;
        if i<3

```



```

        k(i+1,j)=x;
    end
end;
end
% Plot and print commands followed
% END

```

4 NONSOLETOTj+1 equation, n_c_c.m.

```

% A routine to plot the bifurcation diagram
% for a real valued function
% in this case NONSOLETOT = x,
% with a varying parameter = parm, which is used twice.
% It represents the rather complicated equation path from
% NONSOLETOT to the next COMM which is simplified to
% COMM = parm * NONSOLETOT.
% Thus we have the new COMM calculated in the first equation,
% the old COMM was calculated in the previous equation two.
xmin=-2000; xmax=2000; pin=0; pf=2;
nt=8; np=41; xii=1000; xi=0;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph; x1=xii; x0=xi;
    for i=1:nt,
        y=x1+(parm*x0)-(parm*x1); % change this if diff funct
        x0=x1;
        x1=y;
        if y > xmax, y=xmax;
            elseif y < xmin, y=xmin;
        end
        z(i-ni,j)=y;
    end;
end
% Plot and print commands followed
% END

```

5 ECINFRj+1 equations, eh.m, el1.m and er.m.

```

% A routine to plot the behaviour of a real valued function
% in this case ECINFRj+1 = x,
% with varying: CONSUMERFBK = 40, 0 -> 1000,
% L1(V) = 0.2, 0 -> 1,
% H(V) = 0.045, 0 -> 1,
% R(V) = 10000, 1000 -> 40000.
pin=0; pf=.0002; %1
np=101; nt=5; xi=100; ri=10000;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph; x=xi; rs=ri;
    for i=1:nt

```

```

x=x*(1-0.2-parm*rs)+40; %change for diff funct
rs=rs-.05*rs;
if x > 0, y=log10(x); z(i,j)=y;
    elseif x < 0, k=log10(-x); z(i,j)=-k;
    else x == 0, z(i,j)=0;
end
end;
end

```

% The else routine is supposed allow for $x = 0$ but it doesn't
% work - hence this piece of trickery.

```
z(1,61)=0
```

```
% Plot and print commands followed
```

```
% END
```

```
% A routine to plot the behaviour of a real valued function
```

```
% in this case  $ECINFR_{j+1} = x$ ,
```

```
% with varying: CONSUMERFBK = 40, 0 -> 1000,
```

```
%  $L_1(V) = 0.2$ , 0 -> 1,
```

```
%  $H(V) = 0.000011$ , 0 -> 1,
```

```
%  $R(V) = 10000$ , 1000 -> 40000.
```

```
pin=0; pf=1; np=101; nt=100; xi=100; ri=10000;
```

```
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);
```

```
for j=1:np, parm=pin+(j-1)*ph; x=xi; rs=ri;
```

```
    for i=1:nt
```

```
        x=x*(1-parm-0.000011*rs)+40; %change for diff funct
```

```
        rs=rs-.05*rs;
```

```
        if x > 0, y=log10(x); z(i,j)=y;
```

```
            elseif x < 0, k=log10(-x); z(i,j)=-k;
```

```
            else x == 0, v=x; z(i,j)=v;
```

```
        end
```

```
    end;
```

```
end
```

```
% Plot and print commands followed
```

```
% END
```

```
% A routine to plot the behaviour of a real valued function
```

```
% in this case  $ECINFR_{j+1} = x$ ,
```

```
% with varying: CONSUMERFBK = 40, 0 -> 1000,
```

```
%  $L_1(V) = 0.2$ , 0 -> 1,
```

```
%  $H(V) = 0.000011$ , 0 -> 1,
```

```
%  $R(V) = 10000$ , 1000 -> 40000.
```

```
pin=1000; pf=40000; np=40; nt=100; xi=100;
```

```
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);
```

```
for j=1:np, parm=pin+(j-1)*ph; x=xi;
```

```
    for i=1:nt
```

```

x=x*(1-0.2-0.000011*parm)+40; %change for diff funct
parm=parm-0.05*parm;
if x > 0, y=log10(x); z(i,j)=y;
    elseif x < 0, k=log10(-x); z(i,j)=-k;
    else x == 0, v=x; z(i,j)=v;
end
end;
end
% Plot and print commands followed
% END

```

6 Rj+1(V) equations, re.m and rk.m.

```

% A routine to plot the bifurcation diagram
% for a real valued function
% in this case R = x, with varying coefficients,
% ECINFRj(V) with a range of 0 -> 1000, set to 100,
% and K(V) = G(V) + L2(V), parm, with a range of 0 -> 2.
pin=0; pf=250; nt=20; np=101; xi=10000;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=x*(1-.005*parm); % change this definition if diff funct
        if x > 0, y=log10(x); z(i,j)=y;
            elseif x < 0, k=log10(-x); z(i,j)=-k;
            else x == 0, v=x; z(i,j)=v;
        end
    end;
end
% Plot and print commands followed
% END

```

```

% A routine to plot the bifurcation diagram
% for a real valued function
% in this case R = x, with varying coefficients,
% ECINFRj(V) with a range of 0 -> 1000, set to 100,
% and K(V) = G(V) + L2(V), parm, with a range of 0 -> 2.
pin=0; pf=0.012; nt=20; np=101; xi=10000;
ph=(pf -pin)/(np-1);z=zeros(nt-ni,np);

for j=1:np, parm=pin+(j-1)*ph;x=xi;
    for i=1:nt,
        x=x*(1-100*parm); % change this definition if diff funct
        if x > 0, y=log10(x); z(i,j)=y;
            elseif x < 0, k=log10(-x); z(i,j)=-k;
            else x == 0, v=x; z(i,j)=v;
        end
    end;
end;

```

```
end  
% Plot and print commands followed  
% END
```

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